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Simulation of Analytical Calculations When Optimizing Integrative Properties and Composite Metal Coating

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ABSTRACT

In this work, a nonlinear multidimensional regression-tensor (valence 3) model is constructed and investigated for the analytical substantiation of the necessary / sufficient conditions for optimizing the technological calculation of the multifactor physicochemical process of hardening complex composite media of metal coatings. An adaptive-a posteriori procedure for the parametric formation of the target functional of the quality of the integrative physical and mechanical properties of the designed metal coating is proposed. The results of the study can serve as the basic elements of the mathematical language in the creation of automated design of precision nanotechnologies for hardening the surfaces of complex composite metal coatings on the basis of group accounting of multifactorial tribological, as well as anti-corrosion, tests. In this case, the main goal is not so much the formal accuracy of inferences, but rather the clarity of concepts in the development of general problems of tribology associated with precision modeling of nanostructures of complex composite metal coatings. The multivariate regression-tensor model for tribological / anti-corrosion tests is substantiated by means of the least squares identification of multivariate nonlinear regression equations with the minimum tensor norm. This approach, due to the abundance of available computational problems, as well as due to the possibilities that it opens up for applications of nonlinear multivariate regression tensor analysis, can acquire great (extended) significance in the problems of precision multifactorial nonlinear optimization of physicochemical processes. strengthening of complex composite metal coatings and metamaterials.

Keywords: Hardening composite metal coatings, Complex tribological tests, Optimal physical and chemical process, Multivariate regression-tensor analysis

INTRODUCTION

Nonlinear integrative chemical-physical (CP) processes are at the heart of approaches to hardening various surfaces of modern power machines, and this raises questions that are determined by the formulation and investigation of their mathematical models. In this regard, regression analysis models are also of interest, where tensor-regression systems are a sought-after class. These systems, on the one hand, are close to polynomial models

(Draper and Smith 2007) in their predictive properties, admitting analytical description based on tensor calculus (Akivis and Goldberg 1972), functional analysis of strong differentials Frechet (Kolmogorov and Fomin 1976) from nonlinear vector-functions, as well as extreme problem theory. At the same time, they acquire an important role in the nonlinear analysis of multifactorial tribological (Rusanov et al. 2014) and anticorrosion (Rusanov et al. 2012b) properties of complex metal coatings based on mathematical modeling of physical and mechanical (PM) properties of composite media, developing a nonlinear predictive analysis of integrative characteristics of metal coatings induced by the simulated geometry of surface nanostructures (Khomich and Shmakov 2012; Gerasimov et al. 2014).

The mathematical problems of modeling CP processes formulated in the conclusions of papers (Rusanov et al. 2014; 2012b) are developed below. The main goal is not so much the formal accuracy of inferences, but rather the clarity of concepts in the development of general tribology problems (Trukhanov 2013) related to the precision modeling of nanostructures of complex composite metal coatings. In this regard, the problem of formulating the PM functional evaluating the CFF mode of hardening of composite metal coatings is considered. Analytical interpretations of multi-bound conditions for optimization of the CP-mode, under imposed nonlinear (and essentially difficult to formalize) constraints, are constructed (Yakovlev et al. 2012). A multidimensional tensor regression model for tribological/corrosion tests is substantiated by identifying multidimensional nonlinear PM regression equations with a minimum tensor norm using the least mean square (LMS).

Motivations, terminology, and problem statement. Let R a field of real numbers, R^n a vector n-dimensional space over the field R with Euclidean norm $\|*\|_{R^n}$, $\mathrm{col}(w_1, \ldots, w_n) \in R^n$ a column vector with elements $w_1, \ldots, w_n \in R$ and let $M_{n,m}(R)$ the space $n \times m$-matrices with elements of R. Moreover, let T_m^k the space k of -valued covariant tensors (Akivis and Goldberg 1972, p. 61), or, in other words, of real polylinear forms $f^{k,m} : R_1^m \times \ldots \times R_k^m \to R$ with norm $\left\| f^{k,m} \right\|_{T_m^k} := \left(\sum t_{\ldots j \ldots}^2 \right)^{1/2}$. Here $\{t_{\ldots j \ldots}\}$ is the "coordinate matrix" of tensor $f^{k,m}$ with respect to the canonical basis (Horn and Johnson 1989) in the vector space R^m.

Let $v \in R^m$ a vector of varying HF-predictors (Draper and Smith 2007) for the nonlinear PM-regression with a fixed origin in $\omega \in R^m$ (the reference Cf-mode hardening), $w(\omega + v) \in R^n$ is a vector of PC-variable indicators. For the precision description of the multifactor physico-chemical process we consider a multidimensional functional nonlinear system of the $\iota\upsilon\pi\,\upsilon\tau$–$o\upsilon\tau\pi\,\upsilon\tau$ type, which is described by k-valued vector-tensor PC-regression equation of the form:

$$w(\omega + v) = \mathrm{col}\left(\sum_{j=0,\ldots,k} f_1^{j,m}(v, \ldots, v), \ldots, \sum_{j=0,\ldots,k} f_n^{j,m}(v, \ldots, v) \right) + \varepsilon(\omega, v) \quad (1)$$

Function $f_i^{j,m} \in T_m^j$, $\varepsilon(\omega, \cdot) : R^m \to R^n$ is a non-parameterizable vector function of the class

$$\|\varepsilon(\omega, v)\|_{R^n} = o\left(\left(v_1^2 + \cdots + v_m^2 \right)^{k/2} \right), \quad (2)$$

$v = \text{col}\,(v_1, \ldots, v_m)$, $f_i^{0,m}$ 0-rank tensor, representing the tribological index w_i, $i = \overline{1,n}$ PM-quality of the investigated CP process in its reference mode, given by the vector $\omega \in R^m$.

Remark 1. The precision of nonlinear simulation of the CP process in a class of regression-tensor systems (1) (and adaptation of their parameters) is correct because of continuous dependence (Kolmogorov and Fomin 1976, p. 495) of solutions of the differential diffusion equation on its initial boundary conditions. The tensor structure of equation (1) arises in accordance with Theorem 3 (Kostrikin and Manin 1986, p. 255) and the polylinear character (Kolmogorov and Fomin 1976, p. 490) of the Frechet derivatives of higher orders at calculation of the strong differentials at ω from the vector-function $w(\cdot) = \text{col}\,(w_1(\cdot), \ldots, w_n(\cdot))$. Which ultimately summarizes statement 2 of (Rusanov et al. 2012a) (see problem (I) below). The predictive accuracy of the nonlinear PM modelling is represented by the functional estimate (2), as the residual term in the Peano form associated with the exponent k-valence of the tensor equation (1).

The problem of multidimensional nonlinear tensor-regression modeling of the chemical-physical multifactorial metal coating process, which is optimal with respect to a certain "*target criterion*", is set and studied in other papers for the 2-valent model (1). At the same time, analytical solutions of three related methodological positions of this problem of nonlinear optimal mathematical CHF modeling were obtained:

(I) for a fixed vector-predictor $\omega \in R^m$ and its open neighborhood $V \subset R^m$ analytical conditions are defined, under which the vector-function $w(\cdot) : V \to R^n$ of the PC-property indicators satisfies the multidimensional tensor-regression system (1);

(II) obtained a direct algorithm for identifying tensor coordinates $f_i^{j,m}$, $i = \overline{1,n}$, $j = \overline{0,2}$ in the 2-valued tensor-regressive model (1) based on the numerical solution of the LMS problem of a posteriori optimal PC modeling with two criteria:

$$\begin{cases} \min\left(\sum\limits_{l=1,\ldots,q}\left(\left\|w_{(l)} - \text{col}\left(\sum\limits_{j=0,\ldots,k} f_1^{j,m}(v_{(l)}, \ldots, v_{(l)}), \ldots, \sum\limits_{j=0,\ldots,k} f_n^{j,m}(v_{(l)}, \ldots, v_{(l)})\right)\right\|_{R^n}\right)^2\right)^{1/2}, \\ \min\left(\sum\limits_{i=1,\ldots,n}\ \sum\limits_{j=0,\ldots,k}\left\|f_i^{j,m}\right\|_{T_m^j}^2\right)^{1/2}, \end{cases} \tag{3}$$

where $w_{(l)} \in R^n$, $v_{(l)} \in R^m$, $l = \overline{1,q}$ respectively, the vectors of experimental factor-predictors of the CF process, i.e., $w_{(l)}$ is the a posteriori response to the target variation $v_{(l)}$ with respect to the coordinates of the reference vector ω under the condition $\|v_{(l)}\|_{R^m} < 1$ (this inequality is methodologically dictated by condition (2)) q the number of conducted tribological experiments (determined by representativeness of model (1)), carried out taking into account the diffusive multifactor dynamics of CF processes (Kärger et al. 2005);

(III) for the 2-valued tensor-regression model (1) at a given predictor $\omega \in R^m$ and the nominal condition $\varepsilon(\omega, \cdot) \equiv 0$ The analytical solution of the minimization problem, as a nonlinear "v-optimization" of varying (with respect to vector ω) factor-predictors of predictive PC-characteristics for the considered composite metal coatings:

$$\max_{v \in R^m} F(v) := r^T w(\omega + v) = r_1 w_1(\omega + v) + \cdots + r_n w_n(\omega + v), \qquad (4)$$

where the vector-function $v \mapsto w(\omega + v) = \text{col}\,(w_1(\omega + v), \ldots, w_n(\omega + v))$ has a coordinate representation according to the identified LSM model (1)–(3), and $r_i > 0$ weighting coefficients showing the priority of the PC-indicators; it is also possible to study the problem (III) under some $r_j < 0$, which corresponds to the methodological position where in the PC-indicators w_j should be minimized.

The value of nonlinear multifactor tensor-regression analysis is not only in the exact theorems already derived by this method (Rusanov et al. 2014; 2012a), but also in simple and clear heuristic rules (e.g., the condition of experiments $\|v_{(l)}\|_{R^m} < 1$, or equality $n = m$ in consequence 2) involved in the construction of optimal multivariate posterior modeling. Over time, these rules can be brought to the level of strict theorems of regression analysis, but even now their usefulness is undoubted, as experimentally shown in (Rusanov et al. 2014; 2012b) (in (Rusanov 2014) the problem of surface nitriding was considered, and in (Rusanov et al. 2012b) the problem of sulfochromination).

Problem statements (based on the results of (Rusanov et al. 2014; 2012b)):

(i) determine necessary and sufficient conditions of solvability of optimization problem (4) for 3-valued ($k = 3$) functional tensor-regressive system (1);

(ii) construct an algorithm for correction of sufficient conditions of extremum of the stationary point of problem (i) based on r-parametric tuning $r \mapsto r^T w(\omega + v)$ PC-functional

$$v \mapsto F(v) = r^T w(\omega + v). \qquad (5)$$

Optimization of physical and mechanical parameters of the hardening process of metal coatings. Let us consider problem (i) on optimization of the PM characteristics of metal coatings at $k = 3$; note that the solution of the associated multi-criteria problem (II) of parametric identification for $k = 3$ is an uncomplicated modification of statement 3 from (Rusanov et al. 2012a).

In such mathematical statement the nonlinear multivariate prognostic equation (1) can be given in the following vector-matrix-tensor form:

$$w(\omega + v) = c + \text{Av} + \text{col}\,\left(v^T B_1 v + f_1^{3,m}(v, v, v), \ldots, v^T B_n v + f_n^{3,m}(v, v, v)\right) + \varepsilon(\omega, v), \qquad (6)$$

where $c \in R^n$, $A \in M_{n,m}(R)$, $B_i \in M_{m,m}(R)$, $i = \overline{1, n}$. Without loss of generality, we consider that each matrix B_i has an upper triangular structure. The latter circumstance helps to simplify the numerical implementation of the MNC-algorithm (3). Also note that the vector-function $\varepsilon(\omega, \cdot) : R^m \to R^n$ satisfies (according to the functional term (2)) the following qualitative estimate

$$\|\varepsilon(\omega, v)\|_{R^n} = o\left(\left(v_1^2 + \ldots + v_m^2\right)^{\frac{3}{2}}\right).$$

According to (1) in the case of $k = 3$ the considered PM-function of tribological indicators (5) is twice continuously differentiable, that provides (Banach 1972, p. 189) equality of mixed derivatives

$$\partial^2 F(v_1, \ldots, v_m)/\partial v_g \partial v_p = \partial^2 F(v_1, ..., v_m)/\partial v_p \partial v_g \; \forall g, p = \overline{1, m}. \quad (7)$$

Therefore, in the solution of the optimization problem (4) for the 3-valued model (6) the following statement 1 can be considered as the main result according to Theorem 3 in (Kolmogorov and Fomin 1976, p. 505) and Theorem 7.2.5 from (Kärger et al. 2005). But at first let us preliminarily assume that

$$B_i^* := (B_i + B_i^T) \in M_{m,m}(R), i = \overline{1, n},$$

where B_i is the matrix of the system (6) (the matrix of the tensor $f_i^{2,m}$ in the statement, when it is not considered in the system (1) as a symmetric one). In addition, we consider the vector-function

$$v \mapsto \Phi(v) := (r_1 B_1^* + \cdots + r_n B_n^*)^{-1} \, (A^T + [\nabla_v f_1^{3,m}(v, v, v), \ldots, \nabla_v f_n^{3,m}(v, v, v)]) \, \mathrm{r},$$

where $\nabla_v f_i^{3,m}(v, v, v)$ is the gradient of the functional $v \mapsto f_i^{3,m}(v, v, v)$.

Assertion 1. *Stationary points of $v^* \in R^m$ of the problem* (i) *are the solutions of the equation*

$$v^* + \Phi(v^*) = 0. \quad (8)$$

A sufficient condition is that $F(v^*) = \max \{ F(v) : v \in R^m\}$ is the requirement that v^*, as a stationary point of the functional (5), *has an elliptic type. In other words, at the point v^* for the hessian $G(v, r)$ of the functional* (5), *the inequalities*

$$\det[b_{\mathrm{ij}}]_p < 0, \; p = \overline{1, m}, \quad (9)$$

where $[b_{\mathrm{ij}}]_p \in M_{p,p}(R)$, $p = \overline{1, m}$ are the main submatrices of the hessian

$$G(v^*, r) = r_1 \left(B_1^* + [\partial^2 f_1^{3,m}(v, v, v)/\partial v_g \partial v_p|_{v^*}]\right) + \ldots$$
$$+ r_n \left(B_n^* + [\partial^2 f_n^{3,m}(v, v, v)/\partial v_g \partial v_p|_{v^*}]\right) \in M_{m,m}(R),$$

which is equivalent to the fact that the characteristic numbers $\lambda_p(v^*, r)$ of the matrix $G(v^*, r)$ satisfy

$$\lambda_p(v^*, r) < 0, \; p = \overline{1, m}. \quad (10)$$

Corollary 1. *For the variant $k = 2$ the hessian of the functional* (5) *and conditions* (9), (10) *are invariant to the position of the stationary point v^*, the hessian is equal to*

$$G(r) = r_1 B_1^* + \ldots + r_n B_n^*,$$

which leads to the linear dependence of numbers $\lambda_p(r)$, $p = \overline{1, m}$ on the normalization of the vector r.

If $\operatorname{rank}G(r) = m$ *then the solution of equation* (8) *is unique and has the form*

$$v^* = -G^{-1}(r)A^T r,$$

which makes invariant the position of the point v^* *to the normalization of the vector* r.

According to the vector functions $\nabla_v f_i^{3,m}(v,v,v)$ equation (8) is geometrically determined by intersection m quadrics (Kostrikin and Manin 1986, p. 219). In the given statement (essentially geometrical) it is possible to carry out such analysis on the basis of *the fixed point principle* (Kolmogorov and Fomin 1976, p. 75). Then if inequalities (9) (or, equivalently, (10)) are not fulfilled, that is at least one of them has an opposite sign, the stationary point v^* is hyperbolic (saddle point). On the other hand, the change of the inequality < to the reflexive one (i.e. $\operatorname{rank}G(v^*,r) < m$) induces for v^* the structure of a parabolic point. Thus, in the case of a saddle point / parabolic point v^* to ensure its elliptic character (10), a purposeful parametric correction of the functional (5) is required. It is clear that such correction can shift the position of the stationary point, i.e. after this correction a refinement recalculation is required v^* (by virtue of Corollary 1, this recalculation at $k = 2$ in turn does not change the spectrum (10) of the Hessian $G(r)$).

One of the factors affecting the stationary point geometry v^* of assertion 1, is the digital adaptive parametric tuning $r \mapsto G(v^*,r)$, leading to the fulfillment of elliptic conditions (9) or (10) the subject of the next section.

Parametric correction of the PM-functional on the r -parametric family of its hessians. Consider statement (ii): for a stationary point of the optimization problem (i) to construct a numerical procedure for the correction of the weight coefficients $r \in R^n$, assuming satisfaction of spectral conditions (10), i.e. providing elliptic character of the stationary point of v^* of statement 1. This statement is actual in the optimization of v^*-parameters of the Cf-process, when in some target PM-indicators w_j should be optimized (i.e. $r_j < 0$).

Note 2: Since conditions (9) (10) are algebraically equivalent, the use in construction of the adaptive correction $r \mapsto G(v^*,r)$ expansion of determinants (9) is almost inevitably doomed to failure (even by computer algebra tools) due to a large number of terms determined by multivariate regression coefficients.

The conditions of mathematical solvability of the problem similar to (ii) can be obtained only in exceptional cases. In this connection below we shall discuss an approach to this problem based on the ideas of the theory of localization and perturbation of eigenvalues (Kärger et al. 2005). Another useful mathematical tool is the transformation of condition (10) for the problem of "quadratic" stability by constructing a Lyapunov function (Polyak and Shcherbakov 2002, p. 134) (see Conclusion below) in the affine family of hessians of the optimization problem (i) on the grounds that this family depends explicitly on variations of the vector coordinates due to the structure of the function (5) $r \in R^n$.

Let a certain initial vector of $r_0 \in R^n$ of weighting coefficients from statement (ii). For example, heuristic (in particular, neural network) choice of the vector r_0 can be chosen based on equality of its coordinates r_{0i}, $i = \overline{1,n}$ to values of some functions $\Psi_i : R \to R$ (having explicit physical context) that depend on values of functionals $J_i(v) := w_i(\omega + v)$, $i = \overline{1,n}$ from auxiliary tasks of the optimal prediction of the PM-quality by individual target tribological indicators w_i. In particular, at the 2-valued regression model (1), this position, according to corollary 2 from (Rusanov et al. 2012), will be characterized by the following simple proposition.

Assertion 2. *If the maximal valence of tensors k is two, then the vector of* $r_0 = \text{col}\ (r_{01}, \dots, r_{0n})$ *of initial weighting coefficients with coordinates*

$$r_{0i} = \Psi_i(z_i),\ z_i = \max\left\{J_i(v)\colon v \in R^m\right\},\ i = \overline{1,n}$$

has an analytical representation

$$r_0 = \text{col}\left(\Psi_1(c_1 - e_1^T \text{AB}_1^{-1} A^T e_1/2), \dots, \Psi_n\left(c_n - e_n^T \text{AB}_n^{-1} A^T e_n/2\right) \right),$$

where $\{e_i\}_{\ i = \overline{1,n}}$ *the canonical basis in* R^n.

Let us define by $v^0 \in R^m$ some stationary point of the functional (5) in the case when r-priority of sensing points is r_0. Also by $G_0 \in M_{m,m}(R)$ we denote by the hessian of the given functional, calculated for the pair (r_0, v^0) and let

$$G_i := B_i^* + [\partial^2 f_i^{3,m}(v,v,v)/\partial v_g \partial v_p|_{v^0}]\ ,\ i = \overline{1,n}.$$

Then for the admissible linear variation Δr coordinates of the vector $r_0 = \text{col}\ (r_{01}, \dots, r_{0n})$ defined (by virtue of comments to formula (4)) by the area of this variation $W \subset R^n$ of the form

$$\Delta r := \text{col}\ (\Delta r_1, \dots, \Delta r_n) \in W,$$
$$r_i = r_{0i} + \Delta \text{r}_i > 0,\ i = \overline{1,n},$$

Δr-parametric family of linear variations of the hessian $G(v^0, r_0 + \Delta r)$ is defined by a matrix $m \times m$-multivariate of the form:

$$G_0 + \sum_{i=1,\dots,n} \Delta r_i G_i,\ \Delta r \in W. \tag{11}$$

By virtue of (7) the matrices of the family (11) are symmetric.

For the matrices of the manifold (11), the eigenvalues are characterized as a series of optimization problems using the Courant-Fisher theorem (Kärger et al. 2005). Also in the circle of analytic applications of this theorem lie the reasoning of Weyl's theorem (Trukhanov 2013) on the relations between the characteristic numbers of the Hessian G_0 and any matrix from the manifold (11), allowing us to elucidate more transparently the geometric meaning of the following constructions of the linear Δr-correction performed below

$$\Delta r \mapsto (r_0 + \Delta r)^T w(\omega + v)$$

of the target functional (5).

Taking into account the introduced constructions, adaptive tuning of the functional $F(v) = r^T w(\omega + v)$ of tribological quality of the CHF process, providing at variation of the vector $r \in R^n$ at the stationary point, the fulfillment of inequality (10) contains the following statement 3 below. In essence, this statement is an uncomplicated modification (in the strong derivative version $\mathrm{dG}(v^0, r)/\mathrm{dr}|_{r_0}$) on the basis of Theorem 2 (Kolmogorov and Fomin 1976, p. 491) and Theorem 4.1.3 (Kärger et al. 2005), which takes into account the structure of manifold (11) as symmetric matrices.

Assertion 3. *Let* $r = r_0 + \Delta r$, $\left\{ (\lambda_p(r_0), x_p), \ p = \overline{1, m} \right\} \subset R \times R^m$ *is the set of eigenpairs of the hessian* G_0, *i.e.* $\lambda_p(r_0)\, x_p = G_0 x_p$, $p = \overline{1, m}$, *and let, based on the realization of the manifold* (11), *the numbers*

$$g_{\mathrm{pi}} = x_p^T G_i x_p / x_p^T x_p, \quad p = \overline{1, m}, i = \overline{1, n}.$$

Then the eigenvalues of $\lambda_p(v^0, r_0 + \Delta r)$, $p = \overline{1, m}$ *of the hessian* $G(v^0, r_0 + \Delta r)$ *have the form*

$$\begin{aligned} &\lambda_1(v^0, r_0 + \Delta \mathrm{r}) = \lambda_1(r_0) + \textstyle\sum_{i=1,\ldots,n} g_{1i} \Delta \mathrm{r}_i + o\left(\|\Delta \mathrm{r}\|_{R^n} \right), \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\lambda_m(v^0, r_0 + \Delta \mathrm{r}) = \lambda_m(r_0) + {}_{i=1,\ldots,n} g_{\mathrm{mi}} \Delta \mathrm{r}_i + o\left(\|\Delta \mathrm{r}\|_{R^n} \right). \end{aligned} \tag{12}$$

The system (12) gives an estimation of the sensitivity of the hessian spectrum $G(v^0, r_0 + \Delta r)$ to linear variations of Δr_i, $i = \overline{1, n}$ weight coefficients. For nonlinear variations one can refer to the recurrence formulas from (b) (Kostrikin and Manin 1986, p. 154), which can be computed symbolically by means of computer algebra. Of course, this analysis is approximate (it is true for small $\|\Delta r\|_{R^n}$). It is especially efficient for the 2-valued model at $n = m$ (this equality can be realized due to the relative variability of the number of PM-indicators).

Corollary 2. *Let* $k = 2$, $n = m$, $\Lambda(r_0) := \mathrm{col}\ (\lambda_1(r_0), \ldots, \lambda_m(r_0))$ *is a vector of characteristic numbers of the matrix* $(r_{01} B_1^* + \ldots + r_{0m} B_m^*)$ $u\{x_p\}_{p = \overline{1,m}}$ *their respective eigenvectors. Moreover, let* $\Lambda^* := \mathrm{col}\left(\lambda_1^*, \ldots, \lambda_m^*\right)$ *some vector of characteristic numbers, "benchmark/sample" by criterion* (10), *and* $B := [b_{\mathrm{pi}}] m \times m$-matrix *with elements*

$$b_{\mathrm{pi}} = x_p^T B_i^* x_p / x_p^T x_p.$$

Then for $r_0 + \Delta r$ *where the vector of variation has the representation* $\Delta r = B^{-1}(\Lambda^* - \Lambda(r_0))$, *the eigenvalues of the hessian* $G(r_0 + \Delta r)$ *will be* $o(\|\Delta r\|_{R^n})$*-close to the benchmark numbers* $\left\{\lambda_p^*\right\}_{p=\overline{1,m}}$.

Remark 3. Since corollary 2 holds for small $\|\Delta r\|_{R^m}$, it remains an open question whether the iterative computational process will converge to

$$r_j = (r_{j-1} + \Delta r_{j-1}) \in R^m, \ j = 1, 2, \ldots,$$

constructed from the calculation $\Delta r_{j-1} = B^{-1}(\Lambda^* - \Lambda(r_{j-1}))$, if the initial discrepancy $\|\Lambda^* - \Lambda(r_0)\|_{R^m}$ is significant enough. Thus, according to the structure of the target functional (5), at each iteration step j for the vector coordinates $r_j \in R^m$ should be carried out (within the physical setting of problem (4)) the check of coordinate conditions $r_{ij} > 0,\ i = \overline{1, n}$.

Note 4: For adaptive systems, the normalization of input signals (in our case $\|v_{(l)}\|_{R^m} < 1$ in (3)) is essential (this is why adaptive techniques with learning are used). In this context, it is important to obtain (guarantee) sufficient conditions for the adaptive system to have robust bounded solutions (Ackerman 1993), with the very fact of existence of setting solutions satisfying these properties being more important (see (2)) than their specific solutions. Thus, a fixed parameter setting providing a qualitative (see (10)) v-control of the predictive system (1), which is not very sensitive to the exact value of the parameters, can give a range of possible values Δr, making it possible to determine the optimal values v, guaranteeing the target PM-quality (4).

In the context of Remark 3, let us show the result of calculating the upper estimate for the perturbation $\|\Delta r\|_{R^m}$. To this end, let us assume that $\|\bullet\|_M$ is the matrix norm in $M_{m,m}(R)$, consistent with the norm in Euclidean space $\|\bullet\|_{R^m}$, whereby $\|E\|_M = 1, E \in M_{m,m}(R)$ is a unit matrix. For example, the Frobenius norm can serve in this capacity

$$\|D\|_F := (m^{-1} \sum d_{ij}^2)^{\frac{1}{2}},\ D = [d_{ij}] \in M_{m,m}(R),$$

or spectral (induced) matrix norm

$$\|D\|_S := \sup \left\{ \|Dx\|_{R^m} : x \in R^m,\ \|x\|_{R^m} = 1 \right\} = \max_{1 \le i \le m} \lambda_i^{\frac{1}{2}} (D^T D).$$

Let us return to corollary 2: we have $B\ \Delta r = \Lambda - \Lambda(r_0)$, $\det B \neq 0$. Suppose that the vector of characteristic numbers $\Lambda^* - \Lambda(r_0)$ is transformed to the perturbed vector $\Lambda^* - \Lambda(r_0) + \delta$ (in particular, due to the terms of $o\,(\|\Delta r\|)_{R^m}$ of system (12)), and the matrix B is transformed to $B + D$. In this case the vector Δr will obtain (taking into account the modification of consequence 2) some increment θ, transforming to the value of $\Delta r + \theta$ which then will satisfy the following linear algebraic equation

$$(B + D)(\Delta r + \theta) = \Lambda^* - \Lambda(r_0) + \delta.$$

In this case $\delta \in R^m$, $D \in M_{m,m}(R)$ model the perturbations of the vector $\Lambda^* - \Lambda(r_0)$, as well as the inaccuracy of the parametric estimation of the matrix B (if $\|D\|_M \|B^{-1}\|_M < 1$, then $\|D\|_M < \|B\|_M$; see. (Lancaster 1982, p. 197)). Then the calculation of the upper estimate of perturbation $\|\theta\|_{R^m} / \|\Delta r\|_{R^m}$ yields corollary 3. For details of the accompanying (partly routine) calculations using the construction of the conditional number matrix see the popular (among graduate students) monograph (Lancaster 1982, p. 197).

Corollary 3. *Let the assumptions of corollary 2 be supplemented by* $s(B) := \|B\|_M \|B^{-1}\|_M$ *the conditional number of the matrix B where* $\|\bullet\|_M$

is the norm $\|\bullet\|_F$ *or* $\|\bullet\|_S$. *Then the estimation for* θ, $\Delta \mathrm{r}$

$$\|\theta\|_{R^m} \,/\, \|\Delta \mathrm{r}\|_{R^m}\, s(B)\left(1 - s(B)\,\|D\|_M \,/\, \|B\|_M\right)^{-1}$$
$$\left(\|\delta\|_{R^m} \,/\, \left\|\Lambda^* - \Lambda(r_0)\right\|_{R^m} + \|D\|_M \,/\, \|B\|_M\right).$$

If $\|\bullet\|_M = \|\bullet\|_S$ *u* λ_1, λ_m *are, respectively, the smallest and the largest eigenvalues of the matrix* $B^T B$, *then in the last inequality we can assume* $s(B) = (\lambda_m/\lambda_1)^{\frac{1}{2}}$.

Remark 5. The construction of the conditional number $s(B) = (\lambda_m/\lambda_1)^{\frac{1}{2}}$ obtained using the spectral norm $\|\bullet\|_S$, is transparent due to equality $s(B) = \|B\|_S \left\|B^{-1}\right\|_S$.

An alternative to accounting for interference, not only covered by corollary 3, may be approaches, including those involving deep penetration (by means of graphical analysis from works (Rusanov et al. 2014; 2012b)) into the physical content of the subject of nonlinear CF modeling.

CONCLUSION

The purpose of this paper was to develop the analytical results of (Rusanov et al. 2012a) by pointing out the geometrical relation between the problem of determining the value of the matrix hessian function at the stationary point of the target functional (5) and the vector r of weighting coefficients in (5), reflecting the priority between w_i modeled predictions of the target tribological PM indicators. In this context, statement 1 and its corollary 1 show that, unlike the 3-valued in the 2-valued tensor-regression model, the hessian $G(v, r)$ is invariant to the position of the stationary point. In this case, both variants allow us to reveal r-dependence of the hessian spectrum $G(v, r)$ on the basis of a nonlinear multivariate regression PM model for the CFF mode of hardening of composite metal coatings identified within the framework of the LMS problem (II).

Statement 3 essentially asked: what can be said about the eigenvalues of the matrix $G_0 + \sum \Delta r_i G_i$ if each variation Δr_i is a small parameter? Thus, we were interested only in the purely formal aspect of the mathematical modeling problem under study, when we do not consider what should be the real value of the increment Δr_i for the term "small parameter" to be really applicable. In this case the result of statement 3 is based on the assumption that eigenvalues (10) smoothly r-depend on the elements of the hessian $G(v, r)$ during the current parametric r-correction of the target functional (5). However, it should be noted that some (essentially geometric) information is lost in the simulation process when dealing only with the characteristic polynomial, since there are many different matrices with a given characteristic polynomial. Therefore it is not surprising that the stronger results on modeling the hessian spectrum $G(v, r)$, in particular, statement 3 and corollary 2 take into account the structure of $G(v, r)$. The latter allow for technical simplifications by means of specialized computer algebra, based on the geometric position that any hessian matrix is orthogonally similar (Kärger et al. 2005) to a real diagonal matrix.

Numerical methods for finding eigenvalues and eigenvectors represent one of the most important sections of general matrix theory. Previously, no aspect of this topic has been touched upon in the analysis of the vector $\Lambda^* - \Lambda(r_0)$ and the matrix B from consequence 2, but consequence 3 gives an upper bound on the perturbation Δr through the relative perturbations $\Lambda^* - \Lambda(r_0)$, B and the conditional number $s(B)$. Then $s(B)$ is involved in the evaluation in all cases, whether the perturbations occur only in $\Lambda^* - \Lambda(r_0)$, only in B, or in $\Lambda^* - \Lambda(r_0)$ Ð¸ B simultaneously.

Finally, we define another approach (essentially cybernetic) in the adaptive correction $r \mapsto r^T w(\omega + v)$, which is related to the use of sufficient robust stability conditions for the 2-valued model of the matrix $G(r)$, which also leads to conditions (10). In this context, it is necessary that with interval tolerances on the vector coordinates r it is possible to construct the Lyapunov function $V(x) = x_p^T \mathrm{P} \mathrm{x}_p$, where $P \in M_{m,m}(R)$ is a symmetric positively defined matrix, for the latter, the Lyapunov equation $G(r)P + \mathrm{PG}(r) = -Q$ has a solution at a given symmetric positive-defined $m \times m$-matrix Q. The transition to adaptive robust quadratic stability (Polyak and Shcherbakov 2002) and methods for its solution are also proposed in (Ackerman 1993; Kreinovich et al. 1998). This theory, due to the abundance of computational problems available in it and the opportunities it opens for applications of nonlinear multidimensional tensor-regressive analysis, can acquire great (extended) importance in the problems of precision multifactor nonlinear optimization of CP hardening processes of complex composite metal coatings and alloys.

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