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Routing Algorithm and Diameter of Hierarchical Hyper-Star Network

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ABSTRACT

In this paper, we propose a new interconnection network topology, hierarchical hyper-star network *HHS(Cn,Cn)* is based on hyper-star network. Hierarchical networks based on hypercube have been previously proposed; it has been shown that these networks are superior to the basic networks, in terms of various performance including diameter, network cost, fault tolerance etc. Our results show that the proposed hierarchical hyper-star network performs very competitively in comparison to hyper-star network, *HCN(n,n),* and *HFN(n,n)* have been previously proposed. We also investigate a various topological properties of the hierarchical hyper-star network *HHS(Cn,Cn)* including routing algorithm, diameter, connectivity, broadcasting.

Keywords: Interconnection network, Parallel processing, Routing algorithm, Graph

INTRODUCTION

In recent years, the emergence of applied problems in scientific domains which uses large-scale data processing and AI-based models require a high volume of computational cost. Simultaneously, it demands real-time measures to process the series of the input dataset, which advances the milestone of research on parallel processing systems. Parallel computer structure is a high-performance parallel system, which processes large scale data processing and calculation utilizing many connected processors.

The field of parallel processing systems can be subdivided into the following two significant contents: interconnection network design and algorithm development. To categorize these interconnection networks with node number with respect to that was previously suggested, there are Mesh types, Hypercube types, and Star graph types.

Hypercube graph is generally known for its interconnection network of parallel computer structure. Hypercube connection network is widely employed in researches and common used system on account of its merits, which readily provides communications network structure that is demanded in various application field. Hypercube network has simple routing algorithm with node and edge symmetry. Moreover it has maximal fault tolerance, recursive structure, and a forte : it could embed with other interconnection network (Esfahanian et al., 1991. Lee el al., 2002. Leighton, 1992). On the basis of this Hypercube graph, a hierarchical interconnection network *HCN(n,n)* and

HFN(n,n) was proposed. (Abraham, 1991. Duh et al., 1995. Johnsson, 1987. Vaidya, 1993). Hypercube interconnection node has 2^n node and when the number of nodes increments the degree and diameter simultaneously increases proportionally. For an amicable parallel processing, novel Hyper-Star connection network *HS(m,k)* was presented. (Kim and Shin, 1994. Kim et al, 2003). Hyper-Star connection network *HS(m,k)* has regular and irregular structures. Hyper-Star connection network *HS(2n,n)* is a regular structure with degree n. Hyper-Star connection network *HS(2n,n)* retains the properties of the Hypercube and Star graph. Hyper-Star *HS(2n, n)* has similar properties to the Hypercube and the Star graph. When the number of nodes is identical, the Hyper-Star *HS(2n, n)* shows to be superior to Hypercube concerning network cost, and it improved the primary drawback of having an exponentially incrementing number of nodes in a Star graph network. Various research ramifications exist concerning the Hyper-Star graph network, including routing algorithms, Fault Diameter, Bipartite, Broadcasting algorithm, extensibility, fault tolerance, etc. (Lee et al., 2002. Kim et al., 2003).

This research presents a novel Hierarchical HyperStar : HHS connection network graph constructed upon the baseline structure of HyperStar *HS(2n,n)*, and analyses routing algorithm and diameter. Moreover the novel HyperStar HHS graph that this research presents offers distinguished properties in network cost compared to the Hierarchical graph of *HCN(n,n)* and *HFN(n,n)*.

This thesis organization is as follows. In chapter 2 the fundamental characteristics of Hyper-Star network connection will be presented. In chapter 3 we will design the novel Hierarchical Hyper-Star network connection this research proposes, and conduct a thorough analysis of its internal attributes from a theoretical perspective. Furthermore we will compare with the Hyper-Star connection *HCN(n,n)* and *HFN(n,n)* in the perspective of network cost, then destinate to conclusion.

PROPERTIES OF THE HYPER-STAR CONNECTION NETWORK

When the bit-string that indicates the address of the node U of Hyper-Star connection network *HS(m,k)* is composed with total the number of m binary digit, number of "1" is k, and the number of a symbol "0" is constituted with the number of *(m-k)*. Mark this bit-string of node U as $U = 0^{m\text{-}k}1^{k}$.

The total number of nodes in Hyper-Star network *HS(m,k)* is $\binom{m}{k}$, and the number of branches is $(m\text{-}k) \sim k$. This research analyzes the *HS(2n,n)*, which has the equal number of degree in Hyper-Star connection network *HS(m,k)*. Hyper-Star *HS(2n,n)* graph node is expressed through $2n$ bit-string $b_1b_2...b_i...b_{2n}$ and is $|b_i\text{=}``1"|\text{=}|b_i\text{=}``0"|\text{=}n$, $(1 \le i \le 2n)$. In the case of symbol b1 and bi in bit-string are complement, assume σi as a transposition of interchanging b1 and bi. In this circumstance an edge occurs between the two node $u\text{=}b_ib_2...b_1...b_{2n}$ and $v\text{=}b_ib_2...b_1...b_{2n}$. (Figure 1) shows the Hyper-Star *HS(4,2)*.

An arbitrary node *U* constitutes the Hyper-Star connection network *HS*(2*n*,*n*). In the node *U*, the routing path which successively applicates and

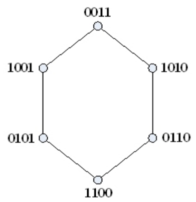

Figure 1: HS(4,2).

goes through the transposition $\sigma_{k1}, \sigma_{k2},..., \sigma_{kt}$, is indicated as $[k_1, k_2,..., k_t]$. For instance, in $HS(4,2)$, a route between node u=0011 and node v=1100 is either (Johnsson, 1987. Duh et al, 1995. Esfahanian et al., 1991).

Suppose $u=u_1u_2...u_i...u_{2n}$ and $v=v_1v_2...v_i...v_{2n}$ as an arbitrary two nodes from Hyper-Star connection network $HS(2n,n)$. Apply Exclusive-OR function between the two nodes u and v and state the results as $R=r_1r_2...r_i...r_{2n}$, $r_i=u_i \oplus v_i$.

$$\text{dist}\,(u, v) = \sum_{i=2}^{2n} r_i \; (r_i = 1)$$

HIERARCHY HYPER-STAR CONNECTION NETWORK DESIGN AND ANALYSIS

The Hyper-Star $HHS(C_n,C_n)$ this thesis suggests is constructed based on Hyper-Star $HS(2n,n)$ as a basic model. Name Hyper-Star $HS(2n,n)$ which composes basic module as 'cluster' and remark it as C_n. The node of the graph $HHS(C_n,C_n)$ is consist of two addresses like as (I,J). I represents the cluster, and J represents an address inside the cluster. The edge of $HHS(C_n,C_n)$ graph is subdivided into n internal edges which are installed in basic module and 1 external edge that connects the basic module. Hierarchical Hyper-Star $HHS(C_n,C_n)$ is composed with the amount of n of 2n combination cluster, and each node attains $n+1$ edges. External edge is classified into a diameter edge and non-diameter edge. Diameter edge is an external edge between the node (I,I) and (J,J). In this circumstance, I and J are complement to each other. An edge that is not in the category of diameter edge is classified as non-diameter edge. (Figure 2) indicates the structure of $HHS(C_2,C_2)$.

ROUTING ALGORITHM

Locate the start node of Hierarchical Hyper-Star $HHS(C_2,C_2)$ as (I,J) and destination node as (K,L). A routing algorithm from start node (I,J) to destination node (K,L) is as follows. If the cluster address of these two nodes are equivalent (that is, $I=K$), apply routing algorithm (Lee et al., 2002) of Hyper-Star graph and represent symbol as $\Rightarrow$. The symbol $\rightarrow$ represents routing between clusters in the routing algorithm below.

1) Routing algorithm A : $(I,J)\Rightarrow(I,K)\rightarrow(K,I)\Rightarrow(K,L)$

2) Routing algorithm B : $(I,J)\Rightarrow(I,L)\rightarrow(L,I)\Rightarrow(L,K)\rightarrow(K,L)$

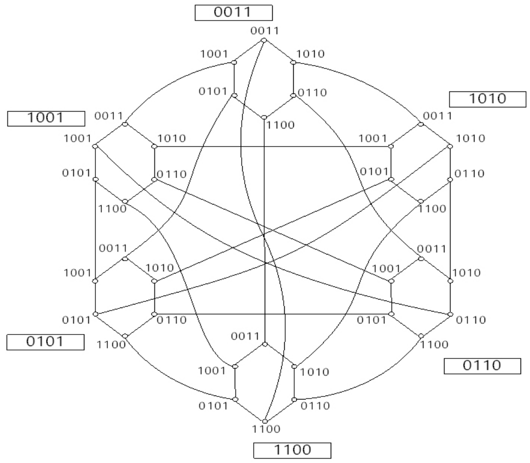

Figure 2: Hierarchical hyper-star graph $HHS(C_2,C_2)$.

3) Routing algorithm C : $(I,J)\Rightarrow(I,M)\rightarrow(M.I)\Rightarrow(M,M)\rightarrow(M',M')\Rightarrow(M',K)\rightarrow(K,M')\Rightarrow(K,L)$

part 1 : $(I,J)\Rightarrow(I,M)\rightarrow(M.I)\Rightarrow(M,M)$

part 2 : $(M',M')\Rightarrow(M',K)\rightarrow(K,M')$

When $M=I$, part 1 is (I,I), and when $M=K'$, part 2 becomes (K,K). Routing distance is as follows.

$R_A = dist(J,K)+dist(I,L)+1$

$R_B = dist(J,L)+dist(I,K)+2$

$R_C = dist(M,I)+dist(M,J)+dist(M',K)+dist(M',L)+\delta$

In the routing distance modification R_C above, value δ is determined as follows. If $M=I=K'$, then $\delta=1$. If $M=I$ or $M=K'(I\neq K')$, then 2, or in every other case is 3. M refers to minimization cluster employed to generate routing distance R_C the shortest distance. M could be found as follows.

Configure cluster M which effects routing distance $Q=dist(M,I)+dist(M,J)+dist(M',K)+dist(M',L)$ become the shortest distance. In this case, when $dist(M',K)=dist(M,K')$, $dist(M',L)=dist(M,L')$, modification attained is as follows :

$Q = dist(M,I) + dist(M,J) + dist(M,K') + dist(M,L') = \sum_{i=2}^{2n}\{(M_i\oplus I_i) + (M_i\oplus J_i)+(M_i\oplus K'_i)+(M_i\oplus L'_i)\}$.

A set of Q-minimization cluster is deducted by discovering the bit string which minimizes the modification $(M_i\oplus I_i)+(M_i\oplus J_i)+(M_i\oplus K'_i)+(M_i\oplus L'_i)$.

First, when $I_iJ_iK'_iL'_i \in\{0111,1011,1101,1110,1111\}$, $M_i = 1$.

Second, when $I_iJ_iK'_iL'_i \in\{0000,0001,0010,0100,1000\}$, $M_i = 0$.

Third, when $I_iJ_iK'_iL'_i \in\{0011,0101,0110,1001,1010,1100\}$, $M_i = X$(0 or 1).

For instance, determine two arbitrary nodes as $(I,J)=(0011,0101)$ and $(K,L)=(0110,1100)$ from Hierarchy Hyper-Star connection network, then bit-strings from $I_iJ_iK'_iL'_I$ becomes 0010, 0100, 1001, 1111. Therefore, $M=00X1$.

As a result the set of Q- minimization cluster is {0001, 0011}, however, the number of bit string 0 and 1 which consists Hyper-Star node is equivalent as n. Consequently every bit string 0 and 1 in every cluster of $HHS(_4C_2,_4C_2)$ must have identical number, and as a sequence set of Q- minimization cluster develop into {0011}, which the number of 0 and 1 corresponds.

Lemma 1 If two random nodes are called (I,J) and (K,L), and Q is a set of Q-minimization clusters, if I(or K') is included in Q_{min} ,only I(or K') is a minimization cluster, otherwise all clusters included in Q_{min} are minimization clusters.

Proof Routing path length of routing algorithm C is $R_C = Q + \delta$. In advance consider the circumstances that is $\neq K'$. If I(or K') is included in Q_{min}, then it is proved that Q and δ are shortest, therefore R_C comes to be the shortest when $M=I$(or K'). As a conclusion I(or K') is a minimization cluster. □

For instance, suppose (I,J)=(000111,101010) and (K,L)=(011001,101010), then the bit string of $I_iJ_iK'_iL'_I$ is 0110, 0001, 0100, 1011, 1110, 1001 with M=$X0011X$. Albeit 4 clusters generate in $Q_{min,}$,

cluster {000111, 100110} is Q_{min} based on the fact that the number of 0 and 1 are equivalent. I

and K' are both minimization cluster because I(=000111) and K'(=100110) come under Q_{min} .

Lemma 2 Suppose arbitrary two nodes (I,J) and (K,L) $(I \neq K)$, and set route P starting from node (I,J) to node (K,L). 5 theorem is established as a result.

2.1 If route P includes over 3 non-diameter edges, route P is not the minimum distance.

2.2 If route P includes over 2 diameter edges, route P is not the minimum distance.

2.3 Routing algorithm A is the shortest path for including one non-diameter edge.

2.4 Routing algorithm B is the shortest path for including two non-diameter edge.

2.5 Routing algorithm C is the shortest path for including one diameter edge.

Proof 2.1 Position (I,J) as source node and suppose destination node as (K,L). Indicate $I=V_0$, $J=V_{-1}$, $K=V_x$, $L=V_{x+1.}$ If route P which connects two nodes contains the number of x non-diameter edge ($x \geq 3$), route P is composed as below.

$P = (V_0,V_{-1}) \Rightarrow (V_0,V_1) \rightarrow (V_1,V_0) \Rightarrow (V_1,V_2) \rightarrow (V_2,V_1) \Rightarrow \ldots \rightarrow (V_x,V_{x-1}) \Rightarrow (V_x,V_{x+1})$.

The routing distance of route P is $R_P = \sum_{i=1}^{x+1} \text{dist}\left(V_{i-2}, V_i\right) = x$

For verification, in this paper we subdivided into two cases depending on the value of x.

(case 1) x is an odd number : Compose route Q, which includes one non-diameter, as follows.

Routing distance of route Q is $R_Q = \sum_{i=1}^{x=1} \text{dist}(V_{i-2}, V_i) + 1$. As a sequence, it is acknowledged that $R_Q < R_P$, the distance of route Q being shorter than route P.

(case 2) x is an even number : Compose route Q which includes two non-diameter edge as follows :

$Q = (V_0,V_{-1}) \Rightarrow (V_0,V_1) \Rightarrow (V_0,V_3) \Rightarrow \ldots \Rightarrow (V_0,V_{x+1}) \rightarrow (V_{x+1},V_0) \Rightarrow (V_{x+1},V_2) \Rightarrow (V_{x+1},V_4) \Rightarrow \ldots \Rightarrow (V_{x+1},V_x) \rightarrow (V_x,V_{x+1})$.

Routing distance of route Q is $R_Q = R_Q = \sum_{i=1}^{x=1} \mathrm{dist}(V_{i-2}, V_i) + 1$. As a sequence, it is acknowledged that $R_Q < R_P$, the distance of route Q being shorter than route P.

Therefore if route P includes over 3 non-diameter edges, it is proved that route P is not the shortest distance.

2.2 Position (I,J) as source node and suppose destination node as (K,L). Set two arbitrary nodes as u and v. If two nodes both exist in the equivalent Hyper-Star and if the route follows algorithm A, the route u==>v signifies a routing path inherent in Hyper-Star.

$P = (I,J) \text{ ==> } (V_1,V_1) \rightarrow (V'_1,V'_1) \text{ ==> } (V_2,V_2) \rightarrow (V'_2,V'_2) \text{ ==> } \ldots \rightarrow \ldots \text{ ==> } (V_x,V_x) \rightarrow (V'_x,V'_x) \text{ ==> } (K,L)$.

Routing distance of route P is as follows.

$R_P = dist(J,V_1)+dist(I,V_1)+ \sum_{i=1}^{x-1} 2\mathrm{dist}(V_i, V_{i+1})+dist(V'_x,K)+ dist(V'_x,L)+\gamma$,

If $\gamma \geq x$, it includes minimum of x diameter edge.

For verification, in this paper we subdivided into two cases depending on the amount of x.

(case 1) when x is an odd number : A path Q including only one diameter edge is configured as follows.

$Q = (I,J) \text{ ==> } (V_1,V_1) \rightarrow (V'_1,V'_1) \Rightarrow (V'_1,V_2) \Rightarrow \ldots \Rightarrow (V'_1,V_{2j}) \Rightarrow (V'_1,V'_{2j+1}) \Rightarrow \ldots \Rightarrow (V'_1,V'_x) \Rightarrow (V'_1,K) \rightarrow (K,V'_1) \Rightarrow (K,V_2) \Rightarrow \ldots \Rightarrow (K,V_{2j}) \Rightarrow (K,V_{2j+1}) \Rightarrow \ldots \Rightarrow (K,V'_x) \Rightarrow (K,L)$.

Routing distance of the route Q is as follows.

$R_Q = dist(J,V_1)+dist(I,V_1)+ \sum_{i=1}^{x-1} 2\mathrm{dist}(V_i, V_{i+1})+dist(V'_x,K)+ dist(V'_x,L)+\gamma$,

If $V_1=I$, γ is 1, if not γ is 2.

When $x \geq 3$, it is acknowledged that $R_Q < R_P$, consequently route Q is in length shorter than route P.

(case 2) x is an even number : Compose a route Q which does not include any diameter edge.

$Q = (I,J) \Rightarrow (I,V_1) \Rightarrow \ldots \Rightarrow (I,V_{2j-1}) \Rightarrow (I,V'_{2j}) \Rightarrow \ldots \Rightarrow (I,V'_x) \Rightarrow (I,K) \rightarrow (K,I) \Rightarrow (K,V_1) \Rightarrow \ldots \Rightarrow (K,V_{2j-1}) \Rightarrow (K,V'_{2j}) \Rightarrow \ldots \Rightarrow (K,V'_x) \Rightarrow (K,L)$.

Routing distance of route Q is as follows.

$R_Q = dist(J,V_1)+dist(I,V_1)+ \sum_{i=1}^{x-1} 2\mathrm{dist}(V_i, V_{i+1})+dist(V'_x,K)+ dist(V'_x,L)+1$,

It is acknowledged that $R_Q < R_P$, consequently route Q is in length shorter than route P.

Therefore, when route P includes over 2 diameter edges, it is acknowledged that route P is not the shortest route.

2.3 The non-diameter edge that connects the two node (I,J) and (K,L) is an arbitrary edge that also links cluster I and K. Thus a route containing one non-diameter edge must necessarily transit an edge that connects two nodes.

A route that accompanies routing algorithm A is an arbitrary route containing one non-diameter edge. Therefore routing algorithm A is a shortest distance among routes that contains one non-diameter edge.

2.4 Form route P including two non-diameter as follows.

$P = (I,J) \Rightarrow (I,V) \rightarrow (V,I) \Rightarrow (V,K) \rightarrow (K,V) \Rightarrow (K,L)$, ($V \neq I$ and $V \neq I$).

Routing distance of route P is as follows.

$R_P = dist(J,V) + dist(I,K) + dist(V,L) + 2$

It is acknowledged that $R_P \geq R_B$, since $R_B = dist(J,L) + dist(I,K) + 2$ and $dist(J,V) + dist(V,L) \geq dist(J,L)$. Therefore routing algorithm B is the shortest distance among routes that includes two non-diameter edges. If $dist(J,V) + dist(V,L) = dist(J,L)$, route P has the same route length as algorithm B and could be replaced as alternative route of B.

2.5 It is readily acknowledged from the definition of algorithm C.

$d = \min(R_A, R_B, R_C)$

According to lemma 2, a regular routing algorithm that connects node (I,J) and (K,L) $(I \neq K)$ could be found in $HHS(C_n,C_n)$. Regular routing algorithm is an algorithm that takes the shortest route among routing algorithm A, B, and C. Distance d, which is the distance between two nodes (I,J)

and (K,L), comes to be the shortest path length among those three path lengths.

$d = \min(R_A, R_B, R_C)$

To examplify, calculate the distance between two node (000111,101010) and (011001,101010). The minimization cluster M between two nodes is 000111. The length of the three routing path is as it shows below.

$R_A = 3+3+1 = 7$.

$R_B = 4+0+2 = 6$.

$R_C = 3+1+2+2 = 8$.

It is acknowledged that routing algorithm B is the regular routing algorithm since R_B has the minimum value.

DIAMETER

Diameter is the maximum value of the shortest distance between the two arbitrary nodes which composes connection network. A delay time occurs because data is conveyed via the whole interconnection network and diameter indicates this lower bound of the delay time.

Theorem 1 The diameter of $HHS(C_n,C_n)$ is $2n + \left\lfloor \frac{2n}{3} \right\rfloor$

Proof Assume source node as (I,J) and destination node as (K,L). In the case of $I=K$, the two nodes exist in the equivalent Hyper-Star and as a consequent the maximum distance is $2n$-1. When $I \neq K$, the distance between two node could be found by the minimum value in three path length $R_A, R_B, R_{C..}$

The three path length can be demonstrated as below.

$R_A = dist(J,K) + dist(I,L) + 1 = \sum_{i=2}^{2n} R_{\mathrm{Ai}} + 1$

$R_B = dist(J,L) + dist(I,K) + 2 = \sum_{i=2}^{2n} R_{\mathrm{Bi}} + 1$

$R_C = dist(M,I) + dist(M,J) + dist(M,K') + dist(M,L') + \delta = \sum_{i=2}^{2n} R_a + \delta$

Table 1. $I_iJ_iK_iL_i$ and R_{Ai}, R_{Bi}, R_{Ci}, M_i.

group	$I_iJ_iK_iL_i$	R_{Ai}	R_{Bi}	R_{Ci}	M_i
1	0000, 1111	0	0	2	$M_i = X$
2	0110, 1001	0	2	2	$M_i = X$
3	0101, 1010	2	0	2	$M_i = X$
4	0011, 1100	2	2	0	$M_i = I_i = k'_i$
5	0010, 1101, 1000, 0111	1	1	1	$M_i = I_i = k_i$
6	0001, 1110	1	1	1	$M_i = I_i \neq k'_i$
7	0100, 1011	1	1	1	$M_i = k_i \neq I_i$

M is minimization cluster, and when $M=I=K'$ $\delta=1$, when $M=I$ or $M=K'(I\neq K')$ it is 2, and in every other case 3. R_{Ai}, R_{Bi}, R_{Ci} can be presented as follows.

$R_{Ai} = (J_i \oplus K_i)+(I_i \oplus L_i)$,

$R_{Bi} = (J_i \oplus L_i)+(I_i \oplus K_i)$,

$R_{Ci} = (M_i \oplus I_i)+(M_i \oplus J_i)+(M_i \oplus K'_i)+(M_i \oplus L'_i)$.

The diameter D of $HHS(C_n,C_n)$ is as follows.

$$D=\max_{(I,J)(K,L)}\{d\} = \max_{(I,J)(K,L)}\{\min(R_A, R_B, R_C)\}.$$

Since it is intricate to express a relationship between three modification with XOR symbol ($\oplus$), as an alternative we will employ plus sign (+) to represent the relationship.

The value of R_{Ai}, R_{Bi}, R_{Ci}, M_i depending on the amount of 4-bit $I_iJ_iK_iL_i(2\leq i\leq 2n)$ was expressed on chart 1. The value of 4-bit $I_iJ_iK_iL_i$, the amount of 2n-1, is subdivided into 7 groups depending on the value of R_{Ai}, R_{Bi}, R_{Ci}, M_i.

Assume b_k as the number of 4-bit value which is in group k. For instance suppose two arbitrary nodes as (000111,010101), (110010,101100), then 5 of 4-bit value is 0110, 0001, 1101, 1010, 1100 which concludes to $b_2=1$, $b_3=1$, $b_4=1$, $b_5=1$, $b_6=1$. Rest of b_k is not existent.

Sigma b_k $(b_1+b_2+\ldots+b_7)$ is $2n$-1. If $b_6=0$, it is known that $M=K'$ and as a consequence $M_i=K'_i$ or $M_i=X$. If $b_7=0$, $M=I$, which leads to a conclusion that $M_i=I_i$ or $M_i=X$ $(2\leq i\leq 2n)$. Therefore if $b_6=0$ or $b_7=0$, $\delta\leq 2$. According to <chart 1>, three routing algorithm path length is indicated as below.

$R_A = 2b_3+2b_4+b_5+b_6+b_7+1$

$R_B = 2b_2+2b_4+b_5+b_6+b_7+2$

$R_C = 2b_1+2b_2+2b_3+b_5+b_6+b_7+\delta$,

In the case of $b_6=0$ or $b_7=0$ it is $\delta\leq 2$, or else it is $\delta=3$.

In this paper we will seperate cases into 4 and calculate diameter according to term of b_k.

(case 1) $b_3\leq b_2$, and $b_4\leq b_1+b_2$.

In this case routing algorithm A is a regular routing algorithm. Distance d is as it shows below.

$d=\min(R_A,R_B,R_C)=R_A=2b_3+2b_4+b_5+b_6+b_7+1\leq 2b_1+4b_2+b_5+b_6+b_7+1$.

It is $b_3=b_2$, and when $b_4=b_1+b_2$ it is proved that $2b_3+2b_4+b_5+b_6+b_7+1 =2b_1+4b_2+b_5+b_6+b_7+1$, and as a consequence $2n$-$1=b_1+b_2+\ldots+b_7=2b_1 +3b_2+b_5+b_6+b_7$. Thus $b_2=(2n\text{-}2b_1\text{-}b_5\text{-}b_6\text{-}b_7\text{-}1)/3\leq\left\lfloor \frac{2n-1}{3} \right\rfloor$, and $2b_1=2n\text{-}3b_2\text{-}b_5\text{-}b_6\text{-}b_7\text{-}1$.

To conclude, it is $R_A\leq 2b_1+4b_2+b_5+b_6+b_7+1=2n+b_2\leq 2n+\left\lfloor \frac{2n-1}{3} \right\rfloor$.

(**case 2**) $b_3\leq b_2$ and $b_4\geq b_1+b_2+1$.

In this specific case routing algorithm C is regular routing algorithm. Distance d is as follows.

$d=\min(R_A,R_B,R_C)=R_C=2b_1+2b_2+2b_3+b_5+b_6+b_7+\delta\leq 2b_1+4b_2+b_5 +b_6+b_7+\delta$.

When $b_3=b_2$ and $b_4=b_1+b_2+1$, it is that $2b_1+2b_2+2b_3+b_5+b_6+b_7+\delta= 2b_1+4b_2+b_5+b_6+b_7+\delta$,

and $2n\text{-}1=b_1+b_2+\ldots+b_7=2b_1+3b_2+b_5+b_6+b_7+1$.

Therefor $b_2=(2n\text{-}2b_1\text{-}b_5\text{-}b_6\text{-}b_7\text{-}2)/3$, and $2b_1=2n\text{-}3b_2\text{-}b_5\text{-}b_6\text{-}b_7\text{-}2$.

If $b_6=0$ or $b_7=0$ it is $\delta\leq 2$, and since $b_5+b_6+b_7\geq 0$ it is $b_2\leq\left\lfloor \frac{2n-2}{3} \right\rfloor= \left\lfloor \frac{2n+1}{3} \right\rfloor$-1.

If not $\delta=3$, it is $b_5+b_6+b_7\geq 2$ therefore $b_2\leq$ $b_2\leq\left\lfloor \frac{2n-4}{3} \right\rfloor=\left\lfloor \frac{2n-1}{3} \right\rfloor$-1.

Therefore it is $R_C\leq 2b_1+4b_2+b_5+b_6+b_7+\delta=2n+b_2+\delta\text{-}2\leq\max(2n+ \left\lfloor \frac{2n+1}{3} \right\rfloor\text{-}1, 2n+) =2n+\left\lfloor \frac{2n-1}{3} \right\rfloor) = 2n + \left\lfloor \frac{2n-1}{3} \right\rfloor$.

(**case 3**) $b_3\geq b_2+1$ and $b_4\leq b_1+b_3$.

In this case routing algorithm B is regular routing algorithm and distance d is indicated as below.

$d=\min(R_A,R_B,R_C)=R_B=2b_2+2b_4+b_5+b_6+b_7+2\leq 2b_1+4b_3+b_5+b_6+b_7$.

When $b_3=b_2+1$ and $b_4=b_1+b_3$, it is that $2b_2+2b_4+b_5+b_6+b_7+2=2b_1+ 4b_3+b_5+b_6+b_7$, and $2n\text{-}1=b_1+b_2+\ldots+b_7=2b_1+3b_3+b_5+b_6+b_7\text{-}1$.

Therefore $b_3=(2n\text{-}2b_1\text{-}b_5\text{-}b_6\text{-}b_7)/3\leq\left\lfloor \frac{2n}{3} \right\rfloor$, and it is $2b_1=2n\text{-}3b_3\text{-}b_5\text{-}b_6\text{-}b_7$.

Accordingly it is $R_B\leq 2b_1+4b_3+b_5+b_6+b_7=2n+b_3\leq 2n+\left\lfloor \frac{2n}{3} \right\rfloor$.

(**case 4**) $b_3\geq b_2+1$ and $b_4\geq b_1+b_3+1$

In this case routing algorithm C is the regular routing algorithm. Distance d is as follows.

$d=\min(R_A,R_B,R_C)=R_C=2b_1+2b_2+2b_3+b_5+b_6+b_7+\delta\leq 2b_1+4b_3+b_5+ b_6+b_7+\delta\text{-}2$.

When $b_3=b_2+1$ and $b_4=b_1+b_3+1$, it is $2b_1+2b_2+2b_3+b_5+b_6+b_7+\delta= 2b_1+4b_3+b_5+b_6+b_7+\delta\text{-}2$, and $2n\text{-}1=b_1+b_2+\ldots+b_7=2b_1+3b_3+b_5+b_6+b_7$. Thus $b_3=(2n\text{-}2b_1\text{-}b_5\text{-}b_6\text{-}b_7\text{-}1)/3$, $2b_1=2n\text{-}3b_3\text{-}b_5\text{-}b_6\text{-}b_7\text{-}1$. If $b_6=0$ or $b_7=0$ it is $\delta\leq 2$, since it is $b_5+b_6+b_7\geq 0$, $b_3\leq\left\lfloor \frac{2n-1}{3} \right\rfloor$.

If else $\delta=3$, since $b_5+b_6+b_7\geq 2$ it is proved that $b_2\leq\left\lfloor \frac{2n-3}{3} \right\rfloor=\left\lfloor \frac{2n}{3} \right\rfloor$-1.

Hence it is $R_C\leq 2b_1+4b_3+b_5+b_6+b_7+\delta\text{-}2=2n+b_2+\delta\text{-}3\leq\max\ \max(2n+ \left\lfloor \frac{2n-1}{3} \right\rfloor, 2n+\left\lfloor \frac{2n-1}{3} \right\rfloor) =2n+\left\lfloor \frac{2n-1}{3} \right\rfloor$.

In this 4 cases the maximum distance is $2n+\left\lfloor \frac{2n}{3} \right\rfloor$, and as aconsequence the diameter of Hierarchical Hyper-Star $HHS(C_n,C_n)$ is $2n+\left\lfloor \frac{2n}{3} \right\rfloor$.

CONCLUSION

This thesis constructs novel Hierarchical Hyper-Star connection network based on Hyper-Star connection network $HS(2n,n)$, and propose routing algorithm. Also this paper analysis network cost through the diameter K of $HHS(C_n,C_n)$. The novel $HHS(C_n,C_n)$ graph this research presents offers distinguished properties in network cost compared to the $HCN(n,n)$, $HFN(n,n)$ graph upon the baseline of Hypercube.

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