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# Comparison of Consumer's Risk Quality for Different Sampling Plan Corresponding to Adjacent Sample Size

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## ABSTRACT

In some practical cases, people want to know the risk difference of the sampling plan corresponding to adjacent sample sizes. This paper mainly discusses the difference of consumer's risk quality for different sampling plan for both the sampling procedures for inspection by attributes and by variables with adjacent sample sizes. What's more, we gave the analysis of consumer's risk quality in different lot size and sample size. The results show that in the sampling procedures for inspection by attributes the exact lot size within the same lot range has little effect to the consumer's risk quality. The key factor affecting the acceptance probability is the acceptance number. In the sampling procedures for inspection by variables, as the sample size increases, the consumer's risk quality decreases.

**Keywords:** Sampling plan, Consumer's risk quality, Lot size, Inspection by attributes, Inspection by variables

## INTRODUCTION

Statistical sampling is a kind of very important and economic means in inspection of product quality. Sampling inspection wants to make the decision about whether to accept the lot from a batch or random samples, about the quality of the batch or process test, which is between without inspection and 100% inspection. In our country, there are nearly 30 national standard about statistical sampling inspection, which formed a relatively complete system. Some of the most representative and most used are GB/T 2828 sampling inspection standard series (GB/T 2828.1-2012, 2012; GB/T 2828.2-2008, 2008; GB/T 2828.3-2008, 2008; GB/T 2828.4-2008, 2008; GB/T 2828.5-2011, 2011) and GB/T 6378 series (GB/T 6378.1-2008, 2008; GB/T 6378.4-2018, 2018). In the procurement of products quality assessment, many enterprises use sampling inspection plan following GB/T 2828.1(Sampling procedures for inspection by attributes-Part 1: Sampling schemes indexed by acceptance quality limit for lot-by-lot inspection). When given lot size inspection level and acceptance quality limit (AQL), one can get the sampling plan (n, Ac, Re)from GB/T 2828.1, where n stands for sample size, Ac stands for the acceptance number and Re stands for the rejection number. However, in the real cases, the sample size we can get maybe just $n - 1$. In other words, when reality conditions restrict that the sample size is not a standard sample size n, namely the sampling plan for (n - 1, Ac, Re) or (n + 1, Ac, Re), the quality of the use of the corresponding risk what's different? This paper mainly discusses the difference of consumer's risk quality for different sampling plan. What's more, we gave the analysis of consumer's risk quality in different lot size.

## STATISTICAL BASIS

Assume we take the sampling plan (n, Ac, Re), we first take out n unit products from the lot as the sample. We use X as the number of the nonconforming products in this sample. Take $P_a(p)$ as the acceptance probability when the actual nonconforming rate is p, then we have

$$P_a(p) = \sum_{d=0}^{\mathrm{Ac}} P(X = d) \tag{1}$$

Assume the lot size is N, in the case of sampling without putting back, X follows hypergeometric distribution, then

$$P_a(p) = \sum_{d=0}^{Ac} \frac{C_{\mathrm{Np}}^{d} C_{N\text{ - }Np}^{n-d}}{C_N^n} \tag{2}$$

Assume the lot size is N, in the case of sampling with putting back, X follows binomial distribution, then

$$P_a(p) = \sum_{d=0}^{Ac} C_n^d p^d (1-p)^d. \tag{3}$$

## COMPARISON OF CONSUMER'S RISK QUALITY FOR DIFFERENT SAMPLING PLAN

### Sampling Procedures for Inspection by Attributes

We mainly compare the consumer's risk quality in different settings: There are two lot size: 68 and 78. We assume that the specified value AQL = 1.5 and use the single sampling plan for normal inspection. Then we can get the sampling plan is (13,0,1). Sometimes we get a sample with sample size 12 or 14. So we consider three sample size: 12,13 and 14. In a word, we consider the following six cases: (1) (68,12,0,1); (2) (68,13,1,2); (3) (68,14,0,1); (4) (78,12,0,1); (5) (78,13,1,2); (6) (78,14,0,1).

For the above six sampling plans, the acceptance probability is as follows separately:

$$P_a(p_1) = \frac{C_{68p_1}^{0} C_{68(1-p_1)}^{12}}{C_{68}^{12}} = \frac{C_{68(1-p_1)}^{12}}{C_{68}^{12}};$$

$$\begin{aligned} P_a(p_2) &= \frac{C_{68p_2}^{0} C_{68(1-p_2)}^{13}}{C_{68}^{13}} + \frac{C_{68p_2}^{1} C_{68(1-p_2)}^{12}}{C_{68}^{13}} \\ &= \frac{C_{68(1-p_2)}^{13}}{C_{68}^{13}} + \frac{68p_2 C_{68(1-p_2)}^{12}}{C_{68}^{13}}; \end{aligned}$$

$$P_a(p_3) = \frac{C^{0}_{68p_3}C^{14}_{68(1-p_3)}}{C^{14}_{68}} = \frac{C^{14}_{68(1-p_3)}}{C^{14}_{68}};$$

$$P_a(p_4) = \frac{C^{0}_{78p_4}C^{13}_{78(1-p_4)}}{C^{13}_{78}} = \frac{C^{13}_{78(1-p_4)}}{C^{13}_{78}};$$

$$P_a(p_5) = \frac{C^{0}_{78p_5}C^{14}_{78(1-p_5)}}{C^{14}_{78}} = \frac{C^{14}_{78(1-p_5)}}{C^{14}_{78}};$$

$$P_a(p_6) = \frac{C^{0}_{78p_6}C^{14}_{78(1-p_6)}}{C^{14}_{78}} + \frac{C^{1}_{78p_6}C^{13}_{78(1-p_6)}}{C^{14}_{78}} = \frac{C^{14}_{78(1-p_6)}}{C^{14}_{78}} + \frac{78p_6C^{13}_{78(1-p_6)}}{C^{14}_{78}}.$$

First we give the operating curve for these samplings plan.

From Figure 1 and Figure 2 we can see that within the same lot range the exact lot size has little effect to the acceptance probability. The key factor affecting the acceptance probability is the acceptance number.

Next we compare the quantitative difference for consumer's risk quality in different settings. Consumer's risk quality is the quality level when the consumer's risk is 10%. Table 2 demonstrates the difference.

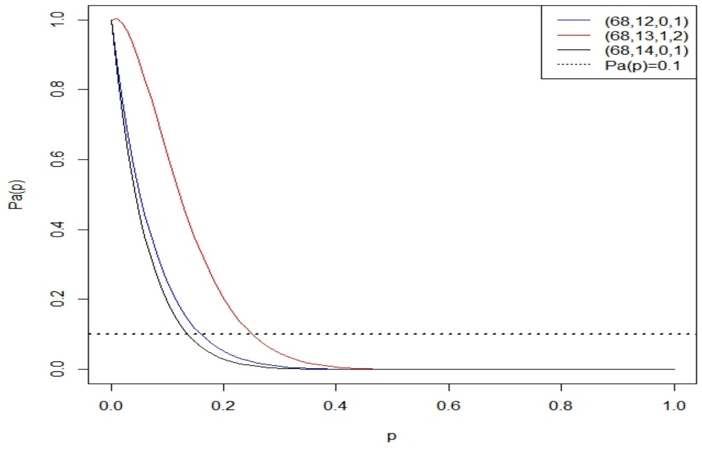


**Figure 1:** The operating curves in lot size 68.

**Table 1.** Consumer's risk quality (CRQ) of different settings.

| Sampling plan | (68,12,0,1) | (68,13,1,2) | (68,14,0,1) |
|---|---|---|---|
| CRQ | 15.99% | 25.08% | 13.65% |
| Sampling plan | (78,12,0,1) | (78,13,1,2) | (78,14,0,1) |
| CRQ | 16.03% | 25% | 13.47% |

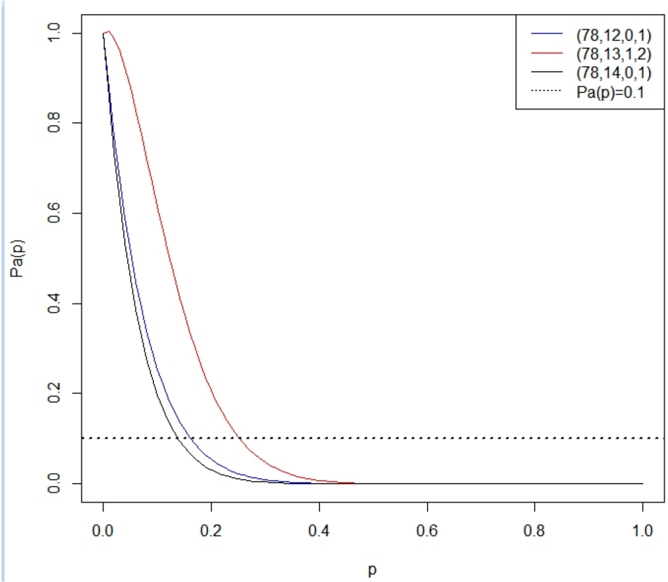


**Figure 2:** The operating curves in lot size 78.

## Sampling Procedures for Inspection by Variables

Sampling procedures for inspection by variables can be divided into two big classes: Standard deviation is known and unknown. We mainly want to compare the CRQ in different sample size. First we consider the case that standard deviation is known.

### Comparison of the Risks of Different Sampling Plans With Known Variance

Assume that the single quality characteristic x follows the normal distribution $N(\mu,\sigma)$, and its the upper specification limit is U, then the nonconforming rate p is as follows:

$$p \;=\; P(X > U) \;=\; 1 - \Phi\left(\frac{U-\mu}{\sigma}\right),$$

then $$\frac{U-\mu}{\sigma} \;=\; \Phi^{-1}(1-p).$$

Assume the acceptance constant is k, then acceptance probability is

$$\begin{aligned} P_a &= P(\overline{x} \le U - k\sigma) = P\left(\frac{\overline{x}-\mu}{\sigma/\sqrt{n}} \le \frac{U-k\sigma-\mu}{\sigma/\sqrt{n}}\right) \\ &= \Phi\left(\frac{U-k\sigma-\mu}{\sigma/\sqrt{n}}\right) = \Phi\left(\sqrt{n}\frac{U-\mu}{\sigma} - k\sqrt{n}\right) \\ &= \Phi\left(\sqrt{n}(\Phi^{-1}(1-p)-k)\right). \end{aligned}$$

When $p = AQL, P_a = 1 - \alpha$, then

$$k = -\Phi^{-1}(AQL) + \frac{1}{\sqrt{n}}\Phi^{-1}(\alpha). \tag{4}$$

We assume $\alpha = 0.05$, AQL = 1, then the different acceptance constant in different sample size is in Table 2:

For every sampling plan (n, k), the consumer's risk is

$$\beta = \Phi(\sqrt{\mathbf{n}}(\Phi^{-1}(\mathbf{1}-\mathbf{p})-\mathbf{k})) \tag{5}$$

When $\beta = 0.1$, we can get consumer's risk quality from (5)

$$\mathbf{p} = \mathbf{1}-\Phi\left(\mathbf{k}+\frac{\Phi^{-1}(\mathbf{0.1})}{\sqrt{\mathbf{n}}}\right). \tag{6}$$

According to (6), we can get the CRQ in different sample size:

From Table 3 we can see that as the sample size increases, the CRQ decreases. For the sake of easy to use, we also can use the same acceptance constant. If we use the same acceptance constant 1.870, then the CRQ in different sample size is shown in Table 4:

From Table 4 we can also see that as the sample size increases, the CRQ decreases. However, there is little decrease comparing to Table 3.

**Comparison of the risks of different sampling plans of double specification limits with unknown variance**

Normally acceptability criteria of double specification limits with unknown variance is due to the acceptance curve. However, GB/T 6378.1 only gives the acceptance curve of the finite type sample size, for example, n = 13, 18... If we want to know the acceptance curve of n = 14, we first need to know the origin of the acceptance curve. Resnikoff (Resnikoff and George, 1952) did a large number of numerical simulations on the different divisions of the

**Table 2.** Acceptance constant in different sample size ($\alpha = 0.05$, AQL = 1%).

| n | 13 | 14 | 15 | 16 |
|---|---|---|---|---|
| k | 1.870 | 1.887 | 1.902 | 1.915 |

**Table 3.** Consumer's risk quality (CRQ) of different settings.

| Sampling plan | (13,1.870) | (14,1.887) | (15,1.902) | (16,1.915) |
|---|---|---|---|---|
| CRQ | 6.49% | 6.13% | 5.81% | 5.54% |

**Table 4.** Consumer's risk quality (CRQ) of different settings (with the same acceptance constant).

| Sampling plan | (13,1.870) | (14,1.870) | (15,1.870) | (16,1.870) |
|---|---|---|---|---|
| CRQ | 6.49% | 6.33% | 6.19% | 6.06% |

nonconforming product rate on both sides, and he found that the differences are so small that they can be regarded as the same OC curve. This drives us to discuss the OC curve of single specification limit.

Assume that the single quality characteristic x follows the normal distribution $N(\mu, \sigma)$. For the upper specification limit U, when the variance is known, it can be deduced that its reception probability is

$$\begin{aligned} \Pr\{\bar{x} + ks \leq U\} &= Pr\left\{\frac{U - \bar{x}}{s} \geq k\right\} \\ &= \Pr\left\{\frac{\frac{\sqrt{n}(U-\mu)}{\sigma} - \frac{\sqrt{n}(\bar{x}-\mu)}{\sigma}}{s/\sigma} \geq \sqrt{n}k\right\} \end{aligned} \tag{7}$$

Denote $T = \frac{\sqrt{n}(U-\mu)}{\sigma} - \frac{\sqrt{n}(\bar{x}-\mu)}{\sigma}$, then it is easily to prove that T follows non-central T distribution with n-1 degree of freedom and $\delta = \sqrt{n}(U-\mu)/\sigma$.

For the upper specification limit U, then the nonconforming rate p is as follows:

$$p = P(X > U) = 1 - \Phi\left(\frac{U - \mu}{\sigma}\right). \tag{8}$$

From (8) we can see that the non-central parameter $\delta = \sqrt{n}\Phi^{-1}(1 - p)$, then the consumer's risk quality p can be solved from the following equation

$$T_{n-1,\sqrt{n}\Phi^{-1}(1-p)}(\sqrt{n}k) = 0.9, \tag{9}$$

where k is the acceptance constant for the single specification limit.

When n = 13, AQL = 1, we can get the acceptance constant 1.712. According to the design guideline of GB/T 6378.1, the different acceptance constants corresponding to different sample sizes are obtained due to the different risks of the corresponding manufacturer. The design principle is that the larger the sample size, the smaller the risk of the manufacturer. The sample size is increased by the design method of approximate preferred number, which implies that when the sample size is 12, 13, 14 or 15, it can correspond to the same producer risk, and therefore, it corresponds to the same acceptance constant. When the sample size is different, the corresponding user risk is different. According to formula (9), given n and k, the corresponding consumer's risk quality of different sample sizes can be calculated as shown in Table 5:

**Table 5**. Consumer's risk quality (CRQ) of different settings.

| Sampling plan | (12,1.712) | (13,1.712) | (14,1.712) | (15,1.172) |
|---|---|---|---|---|
| CRQ | 13.9% | 13.3% | 12.8% | 12.3% |

**Table 6**. Consumer's risk quality (CRQ) of different settings.

| Sampling size | 12 | 13 | 14 | 15 |
|---|---|---|---|---|
| CRQ | 13.9% | 13.3% | 12.8% | 12.3% |

For the unknown variance case with double specification limits, when the sample size is 12, 13, 14 and 15, we can use the same OC curve to give the judgement of acceptance or rejection. However,the acceptance or rejection criteria of different sample sizes correspond to different consumer risks. Conversely, the corresponding consumer risk quality are different for the same consumer risks. The explicit CRQ are as follows:

## CONCLUSION

In this paper, we discusses the difference of consumer's risk quality for different sampling plan both. What's more, we gave the analysis of consumer's risk quality in different lot size both the sampling procedures for inspection by attributes and sampling procedures for inspection by variables. What's more, we gave the analysis of consumer's risk quality in different lot size and sample size. However, there is some limits in this paper. For example, we didn't consider sampling procedures for inspection by variables in non-normal case, which is also our future work.

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