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The Evolution and Evaluation of Pie Menu Interactions in Text Input

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ABSTRACT

Pie menus, also known as radial menus, have been studied in the realm of user interfaces. Various applications of pie menus have been explored, with research focusing on their usage in diverse contexts and their impact on user behaviour. As interactive devices such as touch screen mobiles, virtual reality, mixed reality, and touch-based physical products rapidly permeate into the users' hands, they are driving the demand for studies in multi-model interactions. The pie menu is a promising UI pattern that offers various selection options within a constrained radius. Mostly, the interaction requires human action like click, swipe or steer gestures, and efforts like visual search, decision time. Despite the efforts by different research groups to derive an effective model on human performance, we observed that the derived models on Pie menu are studied independently, but all models are based on Fitts' law and its extension in text input methods. We conducted a comprehensive literature review of publications from ACM, IEEE, HFES, and few others focusing on the evolutions of Pie menus and their derived models. Our qualitative study examined three aspects: application of pie menu in the context of text input, touch mobile interfaces, models applied or derived in these studies. Through a mixed-method analysis, we evaluated these aspects and narrowed our focus to 38 papers that specifically address text input methods and human performance. Our findings revealed that the performance of pie menus varied significantly across different use cases. Certain studies indicate that accessing the northwest side of menus posed greater difficulty, whereas other studies propose that pie menus, although theoretically easy to learn, exhibit less effective performance compared to square menus. However, in the context of text input, pie menus outperformed conventional keyboards. We propose that future research should aim to develop a standard methodology for testing pie menus that can be generalized across different contexts with minimal modifications. The results of this review will provide valuable insights for researchers and designers working on Pie menu interactions, helping them understand previous research, identify gaps, and determine future directions.

Keywords: Pie menu, Radial menu, Menu hierarchy, Review, Fitts' law, Steering law, Narrow tunnel law

INTRODUCTION

With the rapid advancement of technology, interactive devices ranging from traditional devices to mobiles, virtual reality devices to IOT applications had become a part of our daily life of humans (Smith and Johnson, 2021; Chen et al., 2020). This evolution challenges traditional menu system tailored to desktops, necessitating design of intuitive solutions (Lee & Kim, 2020). Researchers now focus on adaptive menu structures, and pie menus are one of the promising solutions where selective options are to be revealed and for their compact, visually intuitive navigation patterns. Pie menus are circular in shape and is divided into sectors or menu item which display command options to choose. Pie menu sometimes are also referred as Radial menu, Circular menu, Polygon menu, or Marking menu. Marking menus sometimes have subtle difference which leaves a track path and the selection targets may not be arranged in equi-distance pattern like in pie menu.

Menu organization can be classified into three types: alpha/numeric, categorical (functional), and random ordering (Dray, Ogden & Vestewig, 1981). Pie menus, which are circular menus that divide the screen into sectors, have been used for all these types of menu organization. One of the innovative applications of pie menu is in its usage in text input methods differently than traditional way of keyboard design, which falls under alpha/numeric category (Callahan et al., 1988; Joshi et al., 2011; Venolia, 1994). Several studies have evaluated the performance and usability of pie menus for text input in different languages, such as English, Hindi, Marathi, etc. (Callahan et al., 1988; Dalvi et al., 2015; Isokoski, 2004; Joshi et al., 2014; Srivastava & Bharath, 2016; Wani et al., 2017).

One of the advantages of pie menus for text input is that they can overcome the challenge of accommodating a large set of characters within the limited space of the keyboard, which is particularly relevant for Indic languages that have complex scripts and multiple variations of characters (Ishida, n.d.). Designing for text input methods is a complex problem and requires huge knowledge on performance models, design, usability, and ergonomics. Previous research on pie menu in text input has been isolated and fragmented, and the methods or models developed have not been systematically applied to link the various research advancements. Pie menu studies have relied on Fitts' law (Barik & Kumar, 2012; MacKenzie, 2013) or empirical experiments and different models are proposed to evaluate the performance in different scenarios. This motivated us to conduct a review study on the use of pie menu in the domain of text input methods. Based on these observations our review is done focused on these research questions:

- What are the specific applications of pie menus within text input methods, and how do they influence user experience and efficiency?
- How does steering law helps in the betterment of the prediction models for the pie menu performances?
- What are the directional effects of steer in the pie menu?

The objective is to study about what are the applications of pie menu, its performance evaluations, methods or models used to measure the performance and provide summary or suggestions according to the limitations and suggested future works identified in this review in the context of text input methods.

METHOD

The methodology of the systematic review adhered to a predetermined review protocol, structured in alignment with the review objectives. This followed the identification of keywords, shortlisting digital libraries, and rigorous screening of literature in accordance with predefined inclusion criteria.

We tried to follow the guidelines provided in Preferred Reporting Items for Systematic Reviews and Meta-Analysis (PRISMA; Liberati et al., 2009). Search for the peer reviewed conference papers and journals on pie menu studies was performed between 10 Jan 2024 to 15 Jan, 2024 in ACM digital library, Springer publisher, IEEExplore a scientific repository, Mendeley search engine, AHFE journals, and Science direct. The review focuses on specific aspects of pie menu application in text input, and smartphones devices and which models are applied in their studies. Relevant keywords were for literature search 'pie menu', 'radial menu', 'marking menu', 'Graphical menu', 'Square menu', 'Radial layout', 'Circular menu', 'Swarachakra', 'chakra menu', 'Flower menu', 'Polygon menu', 'Menu hierarchies', 'Circular keyboard', 'T-Cube' along and in combinations. Few other keywords that were also searched but didn't found papers or produced redundant papers, so they are not extensively used in this review. We presented complete set of information on keywords, filters used, list of papers on pie menu and text input in the Google sheet (http://tinyurl.com/PieSLReview).

We searched same databases for the test input methods studies, using 34 keywords related to text entry, virtual keyboard, and Indic keyboard, and we found around 23 Indian authors, 17 global authors, and 12 Indian organisations and institutes working on this problem. Screened around 900 papers by title and abstract, and found around 200 papers relevant to this context. Out of these, 160 papers focused on English text input methods, and 23 papers focused on Indic text input methods. We also found 17 papers on Fitts' law applied to text input methods (these are not listed in the google sheet).

Figure 1 illustrate the paper count and scrutiny for this review after applying exclusion filters and categorising them into the following themes below.

1. Full circle or half circle Pie menu in Text Input methods in English or Devanagari and using any input method like stylus, mouse, finger, voice and on devices like mobile, desktop, table, smart watch.
2. Pie menu for command or function input using new technologies like AR, VR, Gaze, Air Gesture (but not text input).
3. Pie menu in large screens like Table top, Projections.
4. Pie menu Accessibility, Visual colours.

Additionally, we utilised a visualisation tool Litmaps to depict the momentum of these studies over the time in Figure 2a and 2b.

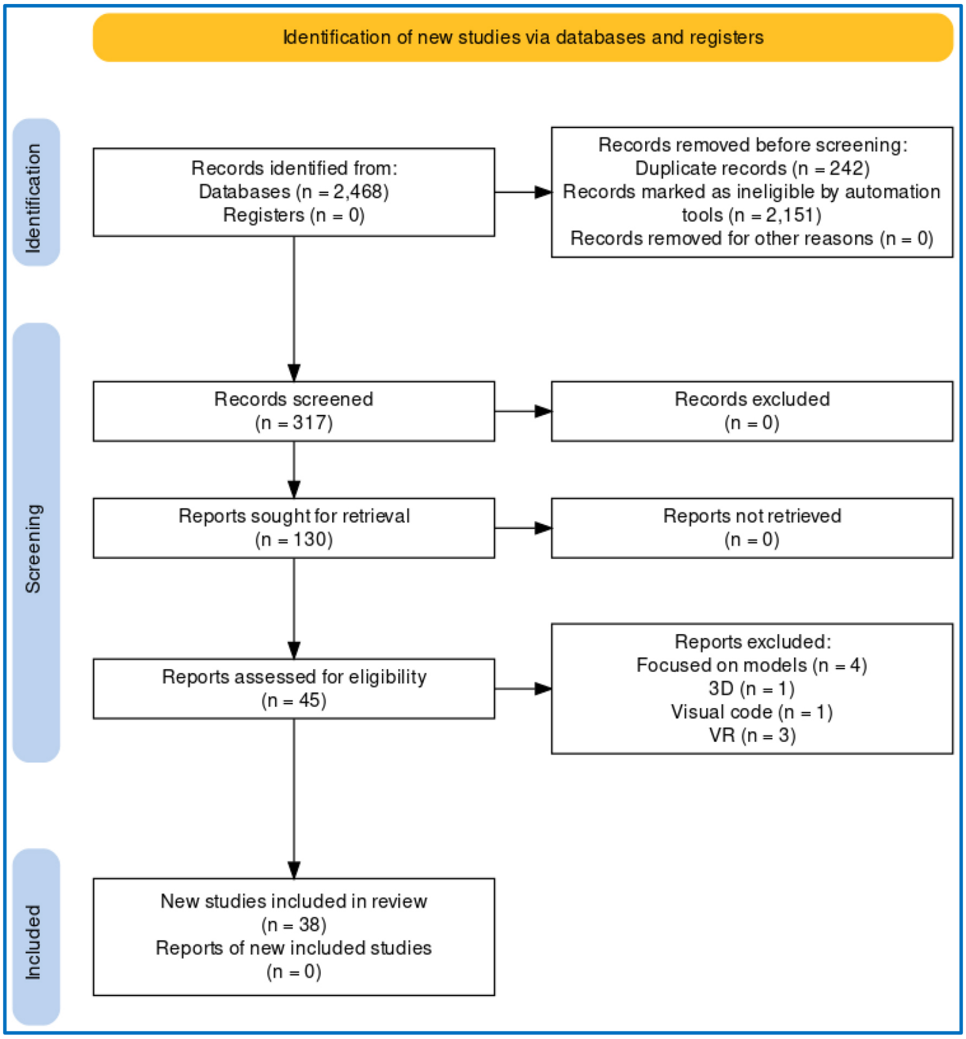

Figure 1: Flow chart of study selection process based on PRISMA 2020.

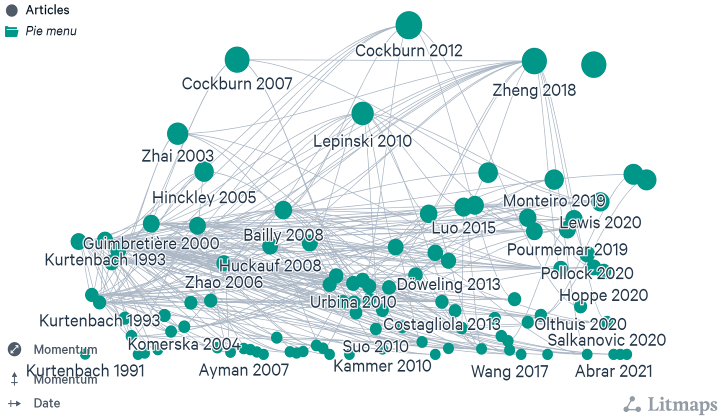

Figure 2: a) Pie menu research momentum vs. date (Tool – Litmaps).

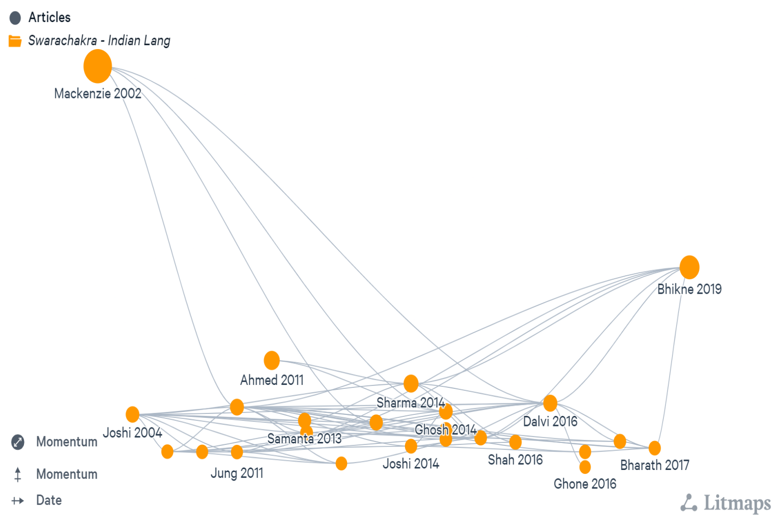

Figure 2: b) Pie menu in Indic text input research (momentum vs. date) (Tool – Litmaps).

Inclusion Criteria

Theme 1 is taken further to study as per the focus of the review, which aims to understand 'input performances', 'directional effect', 'width of pie', 'number of items', 'occlusion', 'handedness', 'form factor', and 'throughput' within the first theme. The study on interactions patterns could be 2D or 3D and models like Fitts' or any new performance model. There no time restriction for collecting the papers, and the publication language is English, of the themes are excluded for this review. Papers which are falling in other themes are not studied.

STUDY

Circular menus, which were first introduced by Neil Wiseman in PIXIE (Wiseman, 1969), have attracted attention for their advantages over linear menus (Callahan et al., 1988) and their implications for Fitts' law and directional effects (Boritz et al., 1991). Notably, pie menus have demonstrated efficacy in touch interfaces and handheld devices (Callahan et al., 1988; Kurtenbach & Buxton, 1993), with users transitioning from novice to expert proficiency with minimal practice (Kurtenbach, 1991). The primary objective of menu design is to optimize movement time, position time, search time, and reaction time. These are explained well in the KLM model (Card et al., 1980) including task selection. Various forms of pie menus, few are shown in Figure 4, have been developed and evaluated, often employing Fitts' law as a robust model ($MT = a + b \times \log2(A / W + 1)$), rooted in Shannon information theory (Fitts, 1954; Shannon & Weaver, 1949). Additional studies have explored methods to enhance performance optimization (Balakrishnan, 2004), typically involving target acquisition through selection (tap) and steering (swipe).

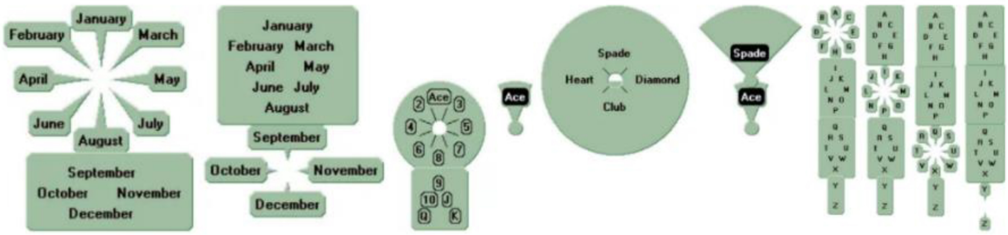

Figure 3: ActiveX pie menu variations (Hopkins, 1998).

Designing effective pie menus necessitates careful consideration of factors such as item quantity, arrangement intuitiveness, semantic grouping, and physical interaction. Suboptimal design or excessive use can undermine their efficacy. Designing pie menus for text input presents further challenges due to the complexity of decision points and the quest for consistency (Anirudha Joshi & Girish Dalvi, 2011; Samanta et al., 2013).

This review focuses specifically on the use of pie menus in text input methods, seeking to synthesize literature findings, identify limitations, and propose potential avenues for future research. We organize our discussion into three thematic areas: the utilization of pie menus in text input, including associated performance models; considerations regarding steering mechanisms; and an examination of directional effects and occlusion factors.

Pie Menus in Text Input

Text entry is often measured with speed and accuracy trade-off. Most of the text entry studies are in English language and studied to find the performance for entering numbers, characters or words or general phrases for text entry methods (Mackenzie & Soukoreff, n.d.). Longitudinal empirical studies were also conducted using onscreen keyboards using desktops or laptops (Callahan et al., 1988) (Dalvi et al., 2015). Evaluated handwriting with recognition software, keypad tapping, pie pad, and moving pie menu.(McQueen et al., 1994) and found that keypad tapping was fastest (30.4 wpm), handwritten (18.5 wpm) whereas pie menu performed the least in both numeric and alphanumeric entry (MacKenzie et al., 1994).

Others studied the implications of menu layout on user performance and understand the differences in performance between menu items placed at different angular positions (Tapia & Kurtenbach, 1995) but didn't focus on text input. Few studies included measured performance using Fitts' but for command selections of menu items with changing mouse positions, and button positions (Lyons et al., 1996). The Chinese PinyinPie augmented the keyboard design and after use of two hours the participants could reach a speed of 25 Chinese characters (Churchill et al., 2012).

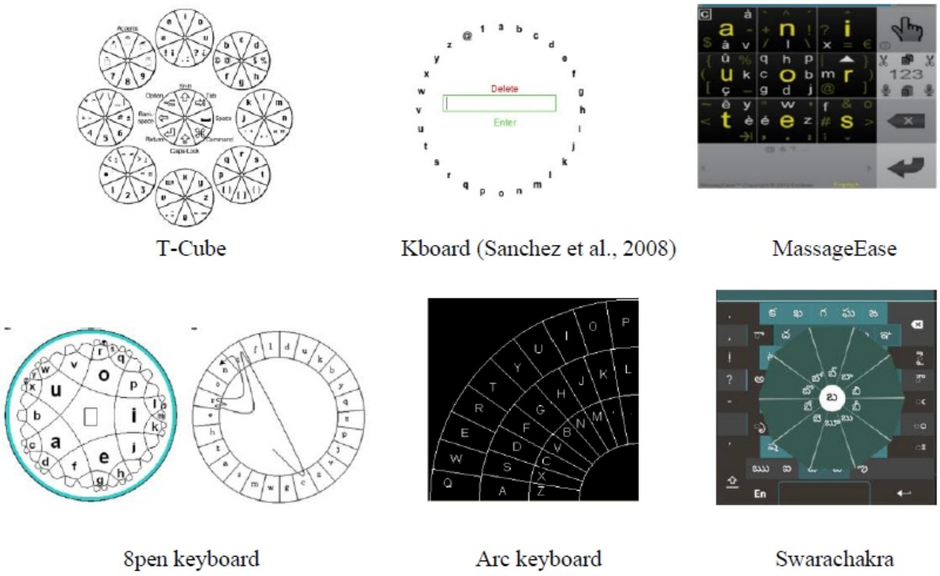

Figure 4: Forms of various pie menu designs.

In the studies conducted by Ahlström et al., (2010), Cockburn et al., (2007), MacKenzie (2013, 2015), and McQueen et al., (1994), models were proposed for predicting the performance of pie menu and steering laws. However, various approaches have been applied in modelling these menus. Some studies used small, medium and large sizes of pie menu, while others varied the numbers of items in the pie from 4 to 49 (Cockburn et al., 2012) (Samp & Decker, 2010). These studies were not conducted in natural context of their use; most of them are lab setups. The most popular forms of pie menus evaluated include SPASE, CRL, Square menu, circular marking menu, circular pie menu, Cirrin, Swarachakra, and FlowMenu. And reported 11–37% of user performance increase by using pie menus in text entry (Isokoski, 2004).

All the papers included in the review utilize models based on Fitts' Law or Steering Law. Some of them do not specifically focus on pie menus, but they still were included in the review because the variations in traditional menu designs suggested by these studies, such as force fields (Ahlström, 2005a), resembled certain design features of radial menus. From the perspective of settings, it is important to emphasize that most studies involve around 12–18 participants with an average age of about 25–28 years old. Interestingly, it is common to engage computer experts in studies on pie menus or at least those individuals who use a computer and a mouse on a daily basis. Only the study by Anirudha Joshi & Girish Dalvi (2011), included novice users with a low level of education. Despite the vast computer experience of participants in most studies, they usually have limited experience with menu items and tools tested by scientists. For instance, the research by Isokoski includes 12 experienced computer users who have never used utilizes marking menus and soft keyboards. The majority of studies seek to predict or report the performance of various menu designs in terms of the speed of selection and radial tasks.

They also offer promising avenues for further research on the problem under investigation.

Unfortunately, studies included in the scope of the review do not use uniform methods to assess the performance of using pie menus in text input methods based on Fitts' Law and Steering Law. Therefore, it is not possible to compare them with each other using a set of consistent indicators. Nonetheless, all of them offer quantitative evidence that could be used to make far-reaching conclusions on the problem under investigation. In particular, the study by Cockburn et al., (2012), found that Fitts' law could effectively explain the performance of different input devices in tapping and dragging tasks. For example, the mean selection time for radial menu selections was 554 ms, but it could be performed faster using the stylus (476 ms). The research by Samp and Decker (2010) found that a hierarchical radial menu performed better than a linear menu in terms of the time of completing pointing (34%) and visual search (14–31%) tasks. Isokoski (2004), found that the introduction of a marking menu could induce an 11–37% improvement in the text entry rate of a soft keyboard. The research by Ahlstrom et al. (2010), demonstrated that the use of square menus in text input methods could display a 15% improvement in performance as compared to linear menus if used by experts. The study by Joshi et al. (2011), offers extensive data on the performance of various menu types. Surprisingly, these scholars did not find empirical evidence for higher text entry rates of different keyboards. The research by Ahlstrom et al. (2006) revealed that the use of a jumping menu, which is a variation of a pie menu, resulted in a 5.6% increase in selection times; furthermore, this increase constituted 20.5% for touchpads. In general, it seems justified to state that the outputs produced in studies included in the scope of this review provide a compelling reason to assert that the use of pie menus in text input methods is associated with a substantial decrease in completion times.

Pie Menu in Indic Language Text Entry

Most studies of pie menu in text input context have used English language, which has only 26 characters, and the conventional keyboards have accommodated these characters according to their frequency of use. However, Indian languages have more than 50 characters on average, and designing a keyboard for these combinations of characters that form glyphs, syllables, conjuncts, and other alpha-syllabary words is very challenging. Traditional Indic keyboards require frequent switching between multiple layers to access these characters.

With the drastic emergence of mobile phones use in India it is important to study and design Indic keyboards for mobile needs. The Swarachakra keyboard designed for touch based mobile phones features a chakra or pie menu, which is a circular menu that allows users to input a combination of consonants and vowel modifiers using a tap and slide gesture. Sparsh, Gboard keyboards uses square menu and popup menu for the same purpose respectively. These menus are designed to ease interaction and reduce cognitive load for users by providing a logical layout and dynamic pop-ups for character input (Anirrudha Joshi & Girish Dalvi, 2011; Joshi et al., 2004; Krishna Dhullipalla, 2012; SIGCHI (Group : U.S.) et al., 2009; Wani et al., 2017). A comparative analysis of Bengali text entry techniques showed that Swarachakra keyboard which uses pie menu for creating glyphs, a gesture-based keyboard, yielded a mediocre performance in terms of speed and accuracy (Sarcar et al., 2017). In a study evaluating Devanagari virtual keyboards for touch screen mobile phones, it was found that the logically structured keyboard, Swarachakra, performed better for first-time usability and in the longitudinal study (Anirrudha Joshi & Girish Dalvi, 2011). Additionally, a study on the performance of Marathi text input mechanisms found that keyboards with logical layouts performed marginally better than keyboards with partially frequency-based layouts and Swarachakra is a logical keyboard in this study and also found that prediction does not improve performance of text input (Dalvi et al., 2016).

The pie menu's role in the Swarachakra keyboard is to provide an intuitive and efficient method for users to input characters and vowel modifiers, enhancing the overall text input experience. Its circular design and tactile feedback contribute to improved target acquisition and touch accuracy, ultimately enhancing the usability of the keyboard for users, including those with visual impairments (Anu Bharath et al., 2017; Bhikne et al., 2019; Srivastava & Bharath, 2016). All the studies on Swarachakra are done very controlled and with a large set of participants, and in natural typing context, so it is important to find cost of the pie menu when compared to other non-pie menu keyboards. This can give some directions on whether this helps in the performance of Indic text input.

Steer Through Pie Menu

The inclusion of steering models in Fitts' law has provided valuable insights into the performance of radial or pie menus. Most of the studies on pie menu has given less attention to dragging or steering phenomenon during item selection. Zhai et al., in their research, noted that the time required for visually guided steering, such as in pie menus, is comparable to or only slightly longer than tapping. This can be quantified using a combination of the steering law and Fitts' law of pointing (Zhai & Kristensson, 2003). We consider the cost of steering actions in pie menus to be significant, and advocate for its inclusion in the assessment alongside movement time, utilizing Fitts' law for pie menu interactions (Ahlström, 2005b). To our knowledge from this review study, we found four studies explored the impact of input devices on radial dragging tasks along with tapping, highlighting the differences in performance based on the type of input device used.

The study by Cockburn et al. (Cockburn et al., 2007, 2012) demonstrated that radial target acquisition is best characterized as a steering activity rather than a Fitts' Law pointing activity. The study adapted the equations for Fitts' Law and the Steering Law to radial selections, providing a new perspective on the movement time for radial selections. The study found that radial dragging is better modelled as a steering activity than as Fitts' Law pointing, indicating that the steering model is more suitable for radial or pie menus. Ahlström et al., (2010) included the steering effect in their proposed model for the multi-level selection of SDP (Search, Decision, and Pointing) times. This model incorporates the concept of ML_i, which represents the sum of individual-level times and steering cost, as defined by Accot & Zhai, (1997), with $sc_j = a+b(A/W + 1)$.

$$ML_i = \sum_{j=1}^{l-1} (T_j + sc_j) + T_l \tag{1}$$

Furthermore, the evaluation of performance metrics encompasses hierarchical pie menus, square menus, and linear menu configurations.

The study conducted by Cockburn et al. (2012), investigates the impact of input devices such as fingers, stylus, and mice on performances across tapping, dragging and radial dragging tasks, with the focus on implications for user experience design in computing devices. The equation 2 for MT_{steer} considers radial target acquisition performance as a linear function of the number of items, incorporating Zhai's steering law (Accot & Zhai, 1997a), with H_N representing the N^{th} harmonic number.

$$MT\mathit{steer} = a + (b/2)\,HN\,Cot(\pi/n) \tag{2}$$

$$H_N = \sum_{i=1}^{n} (1/x) \tag{3}$$

However, there are limitations to the application of steering models in Fitts' law. The study by Nieuwenhuizen and Martens (Nieuwenhuizen & Martens, 2016), highlighted the need for more advanced statistical modelling of experimental data beyond Fitts' law. The study proposed generalizations of Fitts' law to consider the distributions of observed times and to extend Fitts' law to a more general relationship between task characteristics (A/W) and index of difficulty using power law function with an exponent s which is in between 0 and 1. The model is little complex than Fitts' law, however it provides improved data fitting and enhanced differentiation between experimental conditions, which outweighs the drawback of introducing an extra parameter into the model. This suggests that while steering models provide valuable insights, there is a need for more advanced statistical modelling to fully capture the complexities of human performance in selection tasks.

Effect of Directions

All the above studies have used one or more of these mouse cursor, pen and finger. Study using trackball for pie menu selection with six menu items has achieved 3% error rate which is a promising outcome and found that angles nearest to horizontal and vertical directions may have less errors in the pie menu performance (Martin & Isokoski, 2007). The findings on the steering effect showed that the average angular error was 6.7 degrees for the first block and 4.3 degrees for the tenth block. The data indicated that the distribution of angular errors was highly skewed, with a long high-end tail, suggesting that larger menu items were needed to achieve acceptable error rates than what the means would suggest. Additionally, the study demonstrated that after training, the maximum deviations tended to decrease, indicating that using 60-degree menu items could be viable without significant errors.

The study by Hancock et al. found that the movement along the "top-left to bottom-right axis" was fastest for left-handed users, and the mirrored movement along the "top-right to bottom-left axis" was fastest for right-handed users (Brandl et al., 2009). This suggests that the speeds achieved were influenced by the handedness of the users and the occlusion factor. There are few studies found that there is an angle effect for small targets, and short distances (Boritz et al., 1991). Finger movement in North-West direction were more error prone and slower in other directions when compared with pen (Cockburn et al., 2012), this effect may be due to finger tremor when moving. Similar observation was found in Wavelet menus (Francone et al., 2010) which includes South-East marking direction was also found to be slowest and most inaccurate. In few papers it was mentioned the opposite side has same effect in the pie menus. The future directions are to check the performance in nature context of use, upper limits and how this directional effect will be if the sequence shown in opposite way or random way?

IMPLICATIONS AND FUTURE DIRECTIONS

Fitts' law, initially for one-dimensional pointing, was refined for two-dimensional tasks by Accot & Zhai, (2003). The steering law, also by Accot & Zhai (1997a), examines the linear relationship between movement time and difficulty. Both models demonstrate high accuracy, showing reduced navigation time in opposite directions. The theoretical relationship between narrowing and widening steering time was investigated further and found that there is a difference in movement time, so a modified steering law for narrowing tunnel and widening tunnel steer path is derived (Yamanaka & Miyashita, 2016). Figure 5 shows the anatomy of pie menu which is similar to widening tunnel path. The circle represented in the figure is touch point (magnified for our convenience), its diameter depends on the area of the starting target, however in the pie menu sometimes this is almost zero. Our argument is every touch point at the start will not be exactly at the center of the circle, there will be some delta. Similarly, when steered through the pie section, the path of steer will not follow exactly at the axial line of the section, there will be some angle deviation between axis and the steer or stroke, so while modelling the movement time and index of difficulty, we need to ask whether we need to consider time and accuracy trade-off by considering effective width W_e in $ID_e = \log2\ (D/W_e + 1)$, where $W_e = \sqrt{(2\pi e\sigma)} = 4.133\sigma$ (Welford, (1968); MacKenzie, (1992)), will this be applicable to pie menu index of difficulty? Should we consider W(min) (MacKenzie and Buxton (1992)) in case of smartphones where the widths will be in pixels?

How does the steering time and direction vary with the number and angle of pie menu items? How does the use of pie menu affect the text input performance and experience in natural contexts, and what are the upper limits of these effects? What will be the effect of next or before key tap influence on the pie interaction?

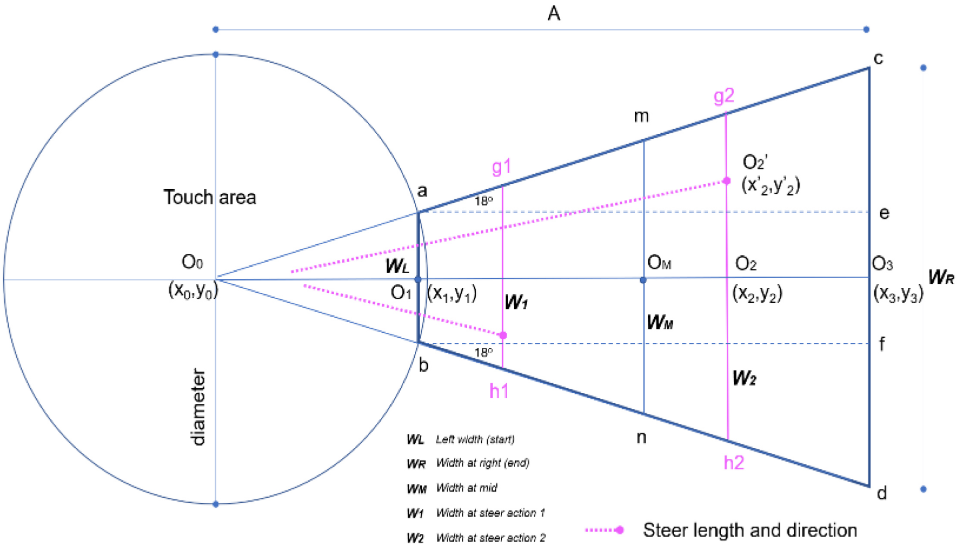

Figure 5: Anatomy of pie section providing various widths at respective steer points.

Considering the above-mentioned gaps in research, which remain unexplored, our ongoing study aims to investigate these factors in the context of pie menus integrated into text input, particularly focusing on smartphone keyboards utilizing Indic scripts. Typing tasks, such as those found in keyboards like Swarachakra (Joshi et al., 2014) when combined with pie menus, involve both pointing and steering actions.

We argue that in text input methods incorporating pie menu interactions, it is essential to consider both pointing and steering actions collectively, as suggested by (Dennerlein et al., 2000; Kulikov, 2006). Drawing from the widening tunnel model (Yamanaka & Miyashita, 2016), pie menu by its shape, which aligns with widening tunnel and with the radial steering angle effect, we propose a model to assess the performance of pointing and steering times in text input context.

$$Tij = \underbrace{\sum_{i=1}^{j} a + b \log_2\left(1 + \frac{A_{ij}}{W_j}\right)}_{\text{Pointing / tapping time}} + \underbrace{c\left(ID_{WTpie}\right)}_{\text{Widening steering tunnel time}} + \underbrace{d\,\left|\sin(\theta)\right|}_{\text{Steering angle factor}}$$

i = starting key
j = target key

Figure 6: Eq. 4 - proposed model include tapping time, widening tunnel steering time, angle factor.

Performance of pointing or tapping is measured through Fitts' model where A is distance from start to end target positions and W is width of the target, and ID_{WTpie} is modified index of difficulty in widening tunnel model (Yamanaka & Miyashita, 2016) (Eq. 5) of a pie section with its absolute direction value $\sin\theta$.

In the Figure 6, W_L is width of the left side of the pie menu which is considered as start position and W_R is width of the target position, which is end width of the pie menu. If we consider the starting position is at the center of the pie menu, then then W_L will be zero, then ID in widening-tunnel model will become infinite. However, in reality, since in the context of text input using mobile phones and tablets with bitmap display, it seems reasonable to consider the minimum value of W_L to be 1px. If W_L =0, then it would mean that finger or cursor is already out of the path at the start of the steering task, leading to an error. So, we derive from the original widening tunnel model -

$$ID_{WT} = (A/(W_L - W_R)) \ln(W_L/W_R) \tag{4}$$

When W_L =1,

$$\mathbf{ID_{WT\mathit{pie}}} = \mathbf{A\ ln((W_R)/(W_R-1))} \tag{5}$$

For example, assuming A = 300 pixels and W_R = 50 pixels, we have ID_{WTpie} = 23.95, which falls within the difficulty range of tasks typically used in steering law experiments. Also, if A = 100 pixels and W_R = 30 pixels are used, we have ID_{WTpie} = 11.73, which appears to be a reasonable value.

If we need to be more accurate in finding W_R and W_L values then, for mouse cursor interaction, we can determine the weighted width W_R or W_L by using Cartesian coordinate data (X,Y coordinates) and apply the method used to find (a+dx) as in this study (MacKenzie, 2018) which will give new A_x, then $\mathbf{W_x = A_x(tan(\pi/n))}$, where 'n' denotes number of pie items. In case of finger touch interaction, assuming a touch area ranging from 6.6mm to 10.6mm (Cockburn et al., 2012), and other research indicating an average touch area of 8mm for finger touch interactions (Dandekar et al., 2003; Joshi et al., 2022; Zhai & Kristensson, 2012), this will be the W_L in the original equation. If the task has only two targets without pie steering or swiping interaction then, ID_{WTpie} will not be used in the Eq. 4.

This study may poise to yield significant empirical insights. Expert users, possess inherent familiarity with steering directions, may experience minimal impact from factors such as reaction time, search time, cognitive processing, or decision-making time, whereas we assume novice users may be more significantly affected by these variables. The considerations become particularly pertinent in contexts involving extensive options, such as text input scenarios. Our future studies are set towards investigating these dynamics.

The future scope lies in refining the steering models and integrating them with advanced statistical modelling to provide a comprehensive understanding of human performance in radial or pie menus. The advancement of technology specially in motion detector sensor devices or technologies like eye gaze, head gaze and motion gaze had made it easy to interact with the different innovative interfaces (Endo et al., 2017; Francone et al., 2010;

Huckauf & Urbina, 2008; Hueber et al., 2020; Kin et al., 2011; Pollock & Teather, 2020; Urbina & Huckauf, 2010; Zeng, 2012). Pie menu is not an exception to be tried out in these new technological innovations. The proposed model can be validated for other contexts as well. Additionally, further research is needed to explore the application of steering models in real-world tasks and to develop more accurate and reliable predictive models for menu performance. This will contribute to the development of more effective and user-friendly menu designs in human-computer interaction.

In summary, while the inclusion of steering models in Fitts' law has advanced our understanding of radial or pie menus, there is a need for more advanced statistical modelling and further research to fully realize the potential of steering models in menu performance analysis.

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