Extending Fitts' Law With Finger Swiping: A Study on Stroke Gesture Dynamics in Commercial Applications

Haotian Ju¹, Fangli Song¹, Wei Wang^{1,2,3}, Jun Zhang¹, Qi Chen⁴, and Le Du⁵

¹School of Design, Hunan University, Changsha, 410000, China ²Lushan Lab, Changsha, 410000, China

³Hunan Provincial Key Laboratory of Intelligent Human Factors Design, Changsha, 410000, China

⁴Guangdong OPPO MOBILE Telecommunications Corp. Ltd., Shenzhen, 518000, China

ABSTRACT

The ubiquity of smartphones and micro-interaction design has underscored the imperative for precise modeling strategies of onscreen gestural behaviors. Traditional research of Fitts' law has quantitatively assessed gestural input efficiency. Fitts' law uses (distance) and (width) to calculate task difficulty. However, it does not fully simulate real-world user behaviors on full screen devices. In this study, we propose an adaptation of Fitts' law to better align with the complex physical dynamics of single-finger stroke gestures. We classified stroke gestures into two Types. To accurately predict and distinguish between them, we developed a modified model introducing three new parameters, initial swiping velocity, end swiping velocity and maximum swiping acceleration. Then we experimented to evaluate our models. The results have proved distinct differences among the two Types of stroke gestures. Empirical data is reported that validates the new models for both Type I and II strokes ($R^2 > 0.9$). We contribute by refining Fitts' law model for real-world use of smartphone applications assisting future gestural interactive design research.

Keywords: Fitts' law, Stroke gesture, Physical modeling, Finger input

INTRODUCTION

With the widespread adoption of smartphones in recent years, multi-touch gestures have played a crucial role in user interactions. How to accurately evaluate the effectiveness of gesture design has become an urgent issue. Fitts' law has been widely used in the evaluation of traditional human-computer interaction interfaces (Barakat et al., 2013). However, for modeling interaction with touchscreens, simply applying Fitts' law may not be accurate enough (Crossman and Goodeve, 1983). Meanwhile, classic Fitts' law and its modified forms are based on typical pointing tasks experiments, but these experiments have significant differences from the current practical findings (Priya and Joshi, 2023). Despite this, the core idea of Fitts' law describing human information input flux still holds. Previous research modifying Fitts'

law on touchscreen achieved results with some limits (Scott Mackenzie, 1992) (Sambrooks and Wilkinson, 2013) (Zhao et al., 2018). Therefore, a reasonable approach is to combine the two-dimensional characteristics of the mobile interface while taking into account the finger movement features of users.

In our study, we analyzed the dynamic characteristics of single-finger swiping gestures and redefined and unified the measurement methods for different types of thresholds. Through an experimental collection of physical data on user operation of single-finger swiping gestures using commercial apps, we evaluated the modification of Fitts' law for single-finger swiping gestures. We hope that this research will enhance Fitts' law to better explain the dynamic characteristics of complex gestures on smartphones, enabling designers and developers to have a deeper understanding of gesture interaction design, and providing new research ideas for other researchers.

Related Research

Fitts' law defines task difficulty (ID) and movement time (MT) to describe the relationship between human performance in manipulating a pointing device and the size of the target or the distance from the target (Paul, 1954). The general mathematical form of one-dimensional Fitts' law is as follows:

$$ID = \left(\frac{D}{W_e} + 1\right), ID = \left(\frac{D}{W_e} + 1\right)$$

A large number of studies have extended the original one-dimensional Fitts law to multidimensional scenarios. It was firstly put forward several different two-dimensional task-oriented rectangular target formulas (Mackenzie, 1992). On this basis, a weighted Euclidean model with a formula to adapt to the influence of proximity angles was proposed (Accot and Zhai, 1997). There are many ways to calculate the W including the area method, heightwidth method, and introducing angle variable (Kopper et al., 2010). A model tailors W to the movement direction towards the target (Jacob et al., 2011). Research indicated that Fitts's law can be used to model purposeful scrolling interactions (Caroline et al., 2006). A search defined target size as the smaller dimension of a 2D shape and compared the usability of Fitts' law on different mobile devices (Sandi et al., 2015). It was also found that experiments on Fitts' law ignore other factors like the arrangement of the buttons and the layout of UI (Priya et al., 2023). A simple linear model based on the concept of constant maximum scrolling speed was proposed (Andersen, 2005). There are also other models proposed to model dynamic revealed targets with different environments, such as handheld devices (Bevan and Fraser, 2016) and multi-touch displays (Zhao et al., 2011). McGuffin and Balakrishnan studied the application of Fitts' law in acquisition tasks, where the difficulty index is calculated from the extended target width (McGuffin and Balakrishnan, 2002). Guiard and Beaudouin-lafon introduced a scale variable and proposed a model that applies Fitts' law by defining a "zoom distance" (Soukoreff and MacKenzie, 2004). A two-handed interaction experiment using a stylus and joystick validated the model. Zhao et al. proposed a modified formula after experimentation to meet the fitting degree of Fitts law for different Types of touch gesture tasks (Zhao et al., 2015).

The thumb's operation on mobile devices is constrained by its three key joints: the Interphalangeal Joint (IPJ), the Metacarpophalangeal Joint (MCPJ), and the Carpometacarpal Joint (CMCJ) (Barakat et al., 2013). And longer thumbs cover a greater area of the screen Ji (Kim and Ji, 2019). Additionally, studies show that the average maximum swiping speed on phones ranges from 25 to 28 cm/s (Bevan and Fraser, 2016). Interaction tasks also affect thumb movement. Conversely, browsing or scrolling tasks may increase thumb movement speed (Karlson et al., 2008). Experienced users demonstrate greater skill and speed in touch screen operations compared to novices (Henze et al., 2011).

The Refined Model of Fitts's Law

With the increasing of interaction design, users can trigger gestures through various methods such as acceleration. Fingers can start moving on the screen or mid-air, which makes calculating complexly. It means that we need to adjust and expand the definition of D and W. We try unify thresholds and stroke gestures. For dynamic parameters, we can treat them as dynamic D and W. D can be defined as the threshold required for acceleration to reach. W can be defined as the range of variation to reach this threshold (Zhao et al., 2011).

Crossman et al. have found that users are more sensitive to the perception of achievable areas (Crossman and Goodeve, 1983), so we define the target width as the threshold range allowed by the system for completing gestures accurately. This definition includes *S*(the achievable screen area), V_0 (the minimum achievable speed), and maxA (the maximum acceleration) allowed by the finger (to reflect the impact of physiological factors on the target width). Finally, *W* can be expressed as: $= s * (V - V_0) * (maxA)$.



Figure 1: Simplified biomechanical modeling of thumb rigid rods.

When executing the swiping gestures, the biomechanical of the thumb's Carpometacarpal (CMC) joint limit its movement speed and range of angles (Kawanishi et al., 2018). These limitations may lead to deviations in thumb movements from the predictions of Fitts' law. When a user holds a phone (see Figure 1), let F_m represent the muscular force, r_m represent the force arm (the vertical distance from the joint to the muscular force application point), F_j represents the joint reaction force, r_j represents the force arm of the joint reaction force, F_e represents the external load (Valero-Cuevas et al., 2003), r_e represents the external load's force arm, then the total torque M_{total} can be expressed as: $M_{\text{total}} = (F_m \cdot r_m) - (F_j \cdot r_j) + (F_e \cdot r_e)$

Swiping gestures with a single finger fall into two categories, distinguished by the presence of a distinct control object and the linearity in the objectfinger movement relationship.

Type I Swiping: Lack a defined control object or linear movement correlation. Users complete these gestures based on innate movement patterns, disregarding set distance or speed thresholds.

Type II Swiping: Involve a clear control object with linear movement correlation. These are characterized by two phases: initially, the finger overcomes static friction and accelerates; subsequently, guided by visual and tactile cues, the finger decelerates.

For rotational motion, Newton's second law can be expressed as $M_{\text{total}} = I \cdot \alpha$. Where M_{total} is the total torque, L is the moment of inertia about the rotation axis, which can be regarded as the rotational mass that depends on the shape and mass distribution of the finger, and α is the angular acceleration. Where r is the distance from the rotation axis to the contact point. The swiping distance of the thumb is related to the angular displacement of the thumb joint. In a rotational system, linear displacement can be calculated by multiplying the rotation radius and the angular displacement. Combining previous expressions we get:

$$d = r \cdot \iint \left(\int \frac{M_{\text{total}}}{I} dt \right) dt$$

From the two formula above, we can find that the nonlinear changes in the total torque of the thumb cause changes in the distance and the angle of finger rotation. This results in even if the values of D and W remain constant, MT still exhibits strong randomness. Therefore, D is redefined as the straight-line distance from the starting point to the endpoint of the user's movement process, and W is defined as $s * v - v_0$) * a, where s is the area of the target region, v_0 is the velocity threshold, v is the user's movement speed, and a is the peak acceleration of the user's movement. Constant total torque sees Type I swiping's movement time positively linked with the reciprocal of ending velocity and acceleration. Conversely, varying torque negates this direct correlation for Type II swiping, prompting the use of average speed instead. Therefore, we use average speed instead of acceleration to describe the movement time of Type II swiping (Brogmus, 1991).

Based on the discussion above, we propose the following research hypothesis: There are two modified Fitts' law formulas for Type I and Type II:

MT =
$$a + b * \log_2(\frac{D}{s}) + c * (\text{endV} - V_0) + d * (\text{maxA})$$

Type II:

MT =
$$a + b * \log_2(\frac{D}{s}) + c * (endV - V_0) + d * (avgV - V_0)$$

In the new modified formulas, there are four parameters a, b, c, and d. We will try to fit the formula through experiments and obtain the values, evaluating our models.

Experiment

Thirty volunteers (15 males and 15 females) participated in this experiment. All participants were right-handed, with 22 aged between 18 and 27 years old, 8 aged between 28 and 42 years old, and an average age of 26.7 years old. The average hand length of male participants was 189.32 millimeters (SD = 8.44), the average palm width was 86.73 millimeters (SD = 6.07), the average thumb length was 58.77 millimeters (SD = 6.73), and the average index finger length was 70.98 millimeters (SD = 4.47). The average hand length of female participants was 174.85 millimeters (SD = 8.55), the average palm width was 75.55 millimeters (SD = 6.16), the average thumb length was 54.65 millimeters (SD = 6.46), and the average index finger length was 65.85 millimeters (SD = 4.22).

The experiment was conducted to simulate everyday scenarios of smartphone usage. The device used in the experiment was an Android phone. Participants completed 10 different task (see Table 1). Each task was repeated 5 times to ensure the reliability of the data.

Tag	Type	Operation Name
1	II	Single Finger Vertical Swipe - Enter app slowly
2	II	Single Finger Vertical Swipe - Enter app quickly
3	II	Single Finger Horizontal Swipe – Switch app once
4	Ι	Single Finger Horizontal Swipe – Fast app switch
5	II	Single Finger Horizontal Swipe – Switch once
6	Ι	Single Finger Horizontal Swipe - Fast switch
7	Ι	Single Finger Vertical Swipe - Exit app
8	Ι	Single Finger Vertical Swipe - Pull down
9	Ι	Single Finger Vertical Swipe - Swipe down on homescreen
10	Ι	Single Finger Vertical Swipe - Swipe down on app

 Table 1. Experimental Operations.

We utilized Android's API to develop a listener program that records gesture data from experimental tasks. This data encompasses finger touch counts, positions, velocity components, and timestamps. The program captures screen feedback, logging it locally. Additionally, several proxy metrics were established to thoroughly detail user tasks (See in Table 2).

Name	Definition	Calculation Method	Unit
startV	The start instantaneous speed	$\sqrt{(V_{x_start})^2 - (V_{y_start})^2}$	pixel/s
endV	The ended instantaneous speed	$\sqrt{(V_{x_end})^2 - (V_{y_end})^2}$	pixel/s
maxV	The maximum instantaneous speed	$v\left(t ight)$	pixel/s
avgV	The average instantaneous speed	$rac{x(t+\Delta t)-x(t)}{\Delta t}$	pixel/s
maxA	The maximum instantaneous acceleration	$\left(\frac{d}{dt}a\left(t\right)\right)$	pixel/s ²
distance	The linear distance between the start and end positions	$\sqrt{(X_1 - X_2)^2 - (Y_1 - Y_2)^2}$	pixel
segTime	The time of completing the touch gesture	Time.end – Time.start	ms

Table 2. Calculation method of processing indicators.

Results Analysis

The ANOVA test yields significant differences on all indicators across Type I (4, 6, 7, 8, 9, 10) and Type II (1, 2, 3, 5) operations (See Table 1), summarized as follows: For 'distance', (F = 46.67, P<.001); 'segTime', (F = 297.15, P<.001); 'startV', (F = 420.15, P<.001); 'endV', (F = 575.66, P<.001); 'maxA', (F = 208.09, P<.001); 'maxV', (F = 244.84, P<.001).

Figure 2 and Figure 3 show the cumulative distribution function (CDF) of endV and avgV of Type I and Type II. By comparing the CDF plots of these two variables, we can find when the total torque is constant, the endV and the $\frac{1}{avgV}$ of the finger motion are positively correlated with the motion time of Type I swiping. In Type I swiping, shorter times correlate with faster endV, explaining the tendency for quicker gestures to larger distances and velocities. Conversely, Type II swiping exhibit a different pattern: changes in total torque don't consistently relate to motion time. This discrepancy likely stems from individual factors. Thus, the $\frac{1}{avgV}$ isn't a reliable measure for predicting Type II swiping times.



Figure 2: CDF of endV.



Figure 3: CDF of avgV.

We tested three fitting algorithms—least squares, gradient descent, and genetic algorithms—to assess their efficacy. Least squares showed limitations with non-linear datasets, while genetic algorithms tended to overfit locally. Consequently, we adopted a combination of gradient descent and least squares for optimizing the modified Fitts' law equation.

The fitting results of the correction formula for single-finger vertical swiping (Type I):



Figure 4: The fitting results of the correction formula for single-finger vertical swiping (Type I).

$$MT = 483 + 272 * log_2 \left(\frac{D}{s}\right) - 20 * (endV - V_0) - 21 * (maxA)$$

The refined model exhibited a high fitness ($R^2 \approx 0.96$), indicating that the model can accurately capture the variations in experimental data. This high degree of explanatory power suggests that the hypothesis of introducing variables endV and maxA has significant predictive value in statistics. The Type I correction formula revealed that the biological force applied by fingers has a decisive impact on the randomness of MT during movement. The user's finger movement pattern is more influenced by the sustained application of biological force, which may lead to non-linear characteristics in the motion process and a tendency to reach a stable speed. Although this stability is achieved in randomness, it still reflects a predictable pattern of movement.

The fitting results of the correction formula for single-finger vertical swiping (Type II):



Figure 5: The fitting results of the correction formula for single-finger vertical swiping (Type II).

$$MT = 1315 + 451 * \log_2\left(\frac{D}{s}\right) - 5 * (endV - V_0) - 107 * (avgV - V_0)$$

The goodness of fit of the modified formula for Type II is still high ($\mathbb{R}^2 \approx 0.91$). The modified formula explains that the MT of Type II singlefinger swiping gesture is more randomly affected by the target area and the distance of finger swiping. Compared with the end speed, the average speed can better reflect the phased finger movement and its subsequent deceleration trend during the entire finger movement process. That is to say, compared with Type I single-finger swiping, the feedback regulation during gesture execution has a far greater impact on MT randomness than the biomechanical characteristics of the finger itself.

DISCUSSION

We found that the trend of finger swiping speed can be used to predict the execution time of Type II single finger swiping. During the finger movement process, the trend of speed change can reflect the difficulty and stage of gesture execution, which can also be used to predict the execution time. Screen size may have a certain degree of impact on experimental results, which is also reflected in the research of Priya et al. (2023) (Tao et al., 2020).

Differences in Type I and II formulas highlight distinct finger motion control mechanisms in multi-touch inputs, illustrating the intricate nature of smartphone interaction performance. This complexity stems from the interplay of physical, biomechanical aspects, and UI design. Type I tasks are governed by inherent factors like finger strength, flexibility, reflecting in freeform gestures like swipes in unbounded spaces. Performance here leans on natural movement patterns and physiological limits. Conversely, Type II tasks emphasize target-directed actions; users adjust trajectories and speeds guided by on-screen elements, like icons. In these cases, average speed and smooth movement are pivotal for efficient, accurate task completion.

In addition, we also found that the area of the target region and the distance of finger swiping can compensate for each other to affect the execution time of Type II single-finger swiping. This means that under the same target area (Okada and Akiba, 2010), the longer the finger swiping distance, the longer the execution time; while under the same swiping distance, the larger the target area, the shorter the execution time (Crossman and Goodeve, 1983). Overall, our research results reveal that the execution time of Type II single-finger swiping is affected by multiple factors, including human factors, the changing trend of finger swiping speed, the area of the target region, and the distance of finger swiping.

CONCLUSIONS

Our research aimed to assess traditional Fitts' law models for describing multi-touch gesture input on mobile devices and propose adjustments for real-world user behavior. We found classic Fitts' parameters insufficient, lacking a predictable link between movement time and target dimensions. Gesture times followed a Poisson distribution, indicative of random influences, with individual biomechanical differences contributing notably to this variability. Consequently, the standard Fitts' model fails to precisely model multi-touch gesture input on mobiles under prevailing design paradigms.

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